

Indicators of the Impact of Changes on the Disposition of Entities and the Morphogenesis of Urban System Connections

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Received: 2023-12-10 | Final version: 2024-12-05

Abstract

The aim of this work is to show the differences in the spatial characteristics of the various dispositions of the entities and the influence of the way of connecting them, on the frequency of their use. The paper presents a methodology that enables interpretation of indicators for a certain number of dispositions of entities from linear, orthogonal and radial-concentric models. The research showed that all the presented variants of the linear disposition retain the same coefficient CC and CL, while CF is significant and decreases where the number of branches connections increases. The orthogonal and radial-concentric disposition show an increase in the coefficient CC depending on the number of current system connections and a rather low coefficient CL, which decreases where the number of connections increases. The same trend is shown by the coefficient CF of orthogonal and radial-concentric disposition, which are significantly lower than in examples of linear disposition. The average index I_{xLACFa} in the orthogonal model has a higher value than in the radial-concentric model due to the geometry of the network, while the average index $I_{xpLACFa}$ has the highest value in the case of the linear model due to the reduction in the frequency of the use by interpolating branch connections. The research methodology shows the possibility of exact calculation of indicators and interpretation of qualitative characteristics of a certain form of disposition of urban system entities at the fourth level of resolution of urban morphology.

Keywords: urban morphology; polycentricity; entities disposition; quantitative indicators

Citation

Petrović-Krajnik, L., Petrović, V., & Krajnik, D. (2025). Indicators of the Impact of Changes on the Disposition of Entities and the Morphogenesis of Urban System Connections. *ACE: Architecture, City and Environment*, 19(57), 12525. <https://doi.org/10.5821/ace.19.57.12525>

Indicadores del Impacto de los cambios en la disposición de las entidades y la morfogénesis de las conexiones del sistema urbano

Resumen

El objetivo de este trabajo es mostrar las diferencias en las características espaciales de las diversas disposiciones de las entidades y la influencia de la forma de conectarlas, en la frecuencia de su uso. El artículo presenta una metodología que permite la interpretación de indicadores para un cierto número de disposiciones de entidades a partir de modelos lineales, ortogonales y radiales-concéntricos. La investigación mostró que todas las variantes presentadas de la disposición lineal conservan el mismo coeficiente CC y CL, mientras que CF es significativo y disminuye cuando aumenta el número de conexiones de ramas. La disposición ortogonal y radial-concéntrica muestra un aumento en el coeficiente CC dependiendo del número de conexiones actuales del sistema y un coeficiente CL bastante bajo, que disminuye cuando aumenta el número de conexiones. La misma tendencia se muestra en el coeficiente CF de disposición ortogonal y radial-concéntrica, que son significativamente más bajos que en los ejemplos de disposición lineal. El índice promedio I_{xLACFa} en el modelo ortogonal tiene un valor mayor que en el modelo radial-concéntrico debido a la geometría de la red, mientras que el índice promedio $I_{xpLACFa}$ tiene el valor más alto en el caso del modelo lineal debido a la reducción en la frecuencia del uso interpolando conexiones derivadas. La metodología de la investigación muestra la posibilidad de cálculo exacto de indicadores e interpretación de características cualitativas de una determinada forma de disposición de las entidades del sistema urbano en el cuarto nivel de resolución de la morfología urbana.

Palabras clave: morfología urbana; policentricidad; disposición de entidades; indicadores cuantitativos

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1. Introduction

The urban system is not only a set of settlements with different dispositions in space, but is also the result of the overall effects of economic, functional, social and infrastructural connections. Urban system entities interact with each other with varying intensities, which, depending on the processes, change over time, affecting the generation and evolution of mutual relations. These interrelationships affect the vertical or hierarchical status of certain entities within the system.

Vresk (2006) emphasizes how urban systems which act as a whole develop over time, and how changes in one element cause changes and "adjustment" of the entire system. Oswald and Baccini (2003) point out that an urban system is a large system made up of geogenic and anthropogenic subsystems. It is an all-encompassing three-dimensional network with different social and physical links.

The network of connections between entities and their disposition in space defines the spatial component of the urban system. The differences in the position of individual entities within the network and their connection with infrastructure networks of a wider area influence the differentiation of the significance of individual entities at the fourth level of urban morphology resolution¹ (Curdes, 1997; Scheer, 2016; Petrović Krajnik et al., 2020). Driven by processes in and between entities at the fourth level of consideration, gradual changes and development of urban systems occur, where monocentric systems turn into polycentric ones, and in some cases when there is no dominance or pronounced hierarchy, the system takes on heteroarchic characteristics (Boix, 2002).

Throughout history, numerous works have dealt with research about characteristics of the spatial organization of entities and the creation of models of urban systems, as well as the processes that characterize them. The increase in the number of the population, the need for agricultural production and the occurrence and spread of industrialization in the 19th century stimulated the development of spatial planning ideas and theories. Pioneering research related to location theory and organizational structure is associated with the economist Johann Heinrich von Thünen, who in his work *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie* (1826) explains the monocentric model of the rural economy with a single market and a large agricultural area that surrounds it and supplies it with the necessary products. He soon presented his Ideal scheme for the arrangement of the land use zones in the "Thünen model" (1842).

German mathematician and economist C.W. Friedrich Launhardt published his research on industrial location theory (1882) and network planning theory (1888). A particularly significant contribution to the study of cities as a system was made by the German geographer Walter Christaller in his work *Die Zentralen Orte in Süddeutschland* (1933), which presents his idea of a central place and centres of different hierarchical rank, which can be seen as the beginning of the idea of polycentrism. The American geographer Charles C. Colby (1933) dealt with the subject of centripetal and centrifugal forces in urban geography, which act between the central zone and the periphery.

The aforementioned works and the changes in the circumstances of the discipline that followed in the 1940s stimulated the appearance of new research on the topic of further development of location theory and studies on industrial locations, central place theory, size and spacing of cities, land use dispositions in cities, "multiple nuclei" theory, locational change and influence of public policies on locational choice, etc., (Lösch, 1940; Ullman, 1941; Chauncy & Ullman, 1945; Hoover, 1948).

¹ At the first level of resolution, the generative elements form a fundamental morphological unit, at the second resolution level they generate the urban tissue texture patterns and urban tissue component networks, and at the third resolution level they generate the composition of the urban structure. The fourth level of resolution involves the positioning of the entities of the urban system.

A significant step forward in considering systems was achieved by Austrian biologist Ludwig von Bertalanffy, founder of general system theory (GST), an interdisciplinary practice that describes systems with interacting components, which is applicable to a variety of disciplines. He questions its impact through systems analysis and systems design (1951, 1968, 1975). The British theorist Stafford Beer (1959) pointed out that systems can be considered on different scales and that there is such a single framework that is a systems view of human relationship with the environment. He sees a system as a set of interconnected parts, and that each part may be seen as a system itself.

Already in the 1960s, Wingo (1961) emphasized the importance of urban transport, which represents the basic spatial organizer of the metropolitan region. The role of transport in the study of spatial planning is also followed by system theories. In those years, spatial planners and professors George Chadwick (1966, 1971) and J. Brian Mc Loughlin (1967, 1969) considered cities and regions as complex wholes with interconnected parts. Mc Loughlin (1969) emphasizes that a system view of cities and regions and also their management can be of great benefit to the practice of planning. It enables us to better understand the extent of certain issues as well as opportunities.

Alongside them, we should highlight other research on the topics of city networks and city systems (Pred, 1977; Ullman, 1980; Dematteis, 1985), network theories (Casti, 1995) and the multidimensionality of networks of complex systems (Johnson, 1995; Allen, 1997). Davoudi and Pendlebury (2010) point out that spatial problems should be framed as scientific problems using systems theory, articulated through interaction models, and solved through the science of system analysis and control.

On the basis of previous theories and quantification and qualification of the characteristics of various networks of urban systems, two basic models of spatially functional organization stand out: monocentric and polycentric. Many authors find differences between the two models by analysing various variables and the influence on their characteristics. Bertaud (2004) and Schwanen et al. (2004) analyse the impacts of monocentric and polycentric models of urban systems and consider that the monocentric model is a recipe for reducing the length of trips and reducing the clutter caused by traffic. Davoudi (2010) emphasizes the role of polycentrism as a key component of the integrated strategy of European spatial development, tries to clarify multiple interpretations of the concept of polycentrism at different spatial levels and considers polycentric urban regions as a means for economic competitiveness and reducing regional differences in Europe.

Meijers (2005), relying on the theory of economic networks, considers polycentrism as a synergistic effect of urban system entities whose whole is greater than the sum of their parts, and Lambregts (2006) considers a possible counterweight to the sometimes overly enthusiastic belief that polycentrism represents a solution to urban and regional problems, and points out that it is necessary to consider the diversity of the morphological origin of the emergence of polycentric urban structures. Meijers and Sandberg (2008) question whether polycentric development is the key to reducing regional differences and points out the necessity of critical reflection on the value of polycentric development as a concept for achieving cohesion.

Trullén and Boix (2008) on the example of the metropolitan region of Barcelona, as one of the most interesting examples of cultural knowledge and a creative metropolis, argue that its territorial expansion did not result from a process of hierarchical decentralization, but rather it represents the effect of increased interaction between the urban continuum of Barcelona and the surrounding medium-sized industrial cities.

Gordon and Richardson (1996) note on the example of the metropolitan area of LA in the USA that "after polycentrism" the urban form manifests itself in the form of *generalized dispersion*. Roca Cladera et al. (2009) points out that centres and sub-centres form a metropolis as *a city of cities*, therefore the metropolitan area can be considered as one that consists of "urban sub-systems, characterized by greater or lesser degrees of monocentrism, polycentrism or dispersion" (p. 2842).

ESPON (2020) in its project *SUPER: Sustainable urbanization and land-use practices in European Regions* highlights three possible forms of the urban expansion model, compact, polycentric and diffuse, and concludes that there are no unique solutions and that it is difficult to make overall judgments about sustainability on a pan-European level because the distribution of development is very heterogeneous.

Fujita et al. (1999) deal with the endogenous formation of hierarchical urban systems, propose an evolutionary approach to system development that connects the general equilibrium model with the dynamics of adaptation, and show the conditioning of the development of the urban system *a la Christaller* and the gradual increase of the economic population. In the work *Urban Structure and Polycentrism: Towards a Redefinition of the Sub-centre Concept* Roca Cladera J. et al. (2009) deal with the fourth level of resolution of urban morphology, defining a subcentre as a structural element of an urban subsystem within the configuration of a metropolitan area with greater or lesser features of polycentrism.

Burger and Meijers (2011) point out that prior research is still scarce and with conflicting results, leaving room for new research. This thesis is supported by Boix and Trullén (2011), who stated in their work that in some environments the positive effects of polycentrism can be seen, while in others they are declared negative, noting that the global effects have never been quantified, which does not mean that urban structures and their form have no effect on man, the organization of the economy and the environment.

Marmolejo et al. (2015) explain the essential differences and roles of polynuclear and polycentric in a functional sense. They see polycentrism as a process that interactively connects several (poly) centres with each other, establishing respective complementarities and the interaction of sub-centres with the environment, forming subsystems, and emphasizing the need for orbital relations between them. In the paper, they combine various indicators to lay the foundation for an integrated measure of polycentrism. They raise and answer the question of whether polyfunctionality, or polycentricity is a desirable aspect, while at the same time offering that the answer depends on what is being analysed.

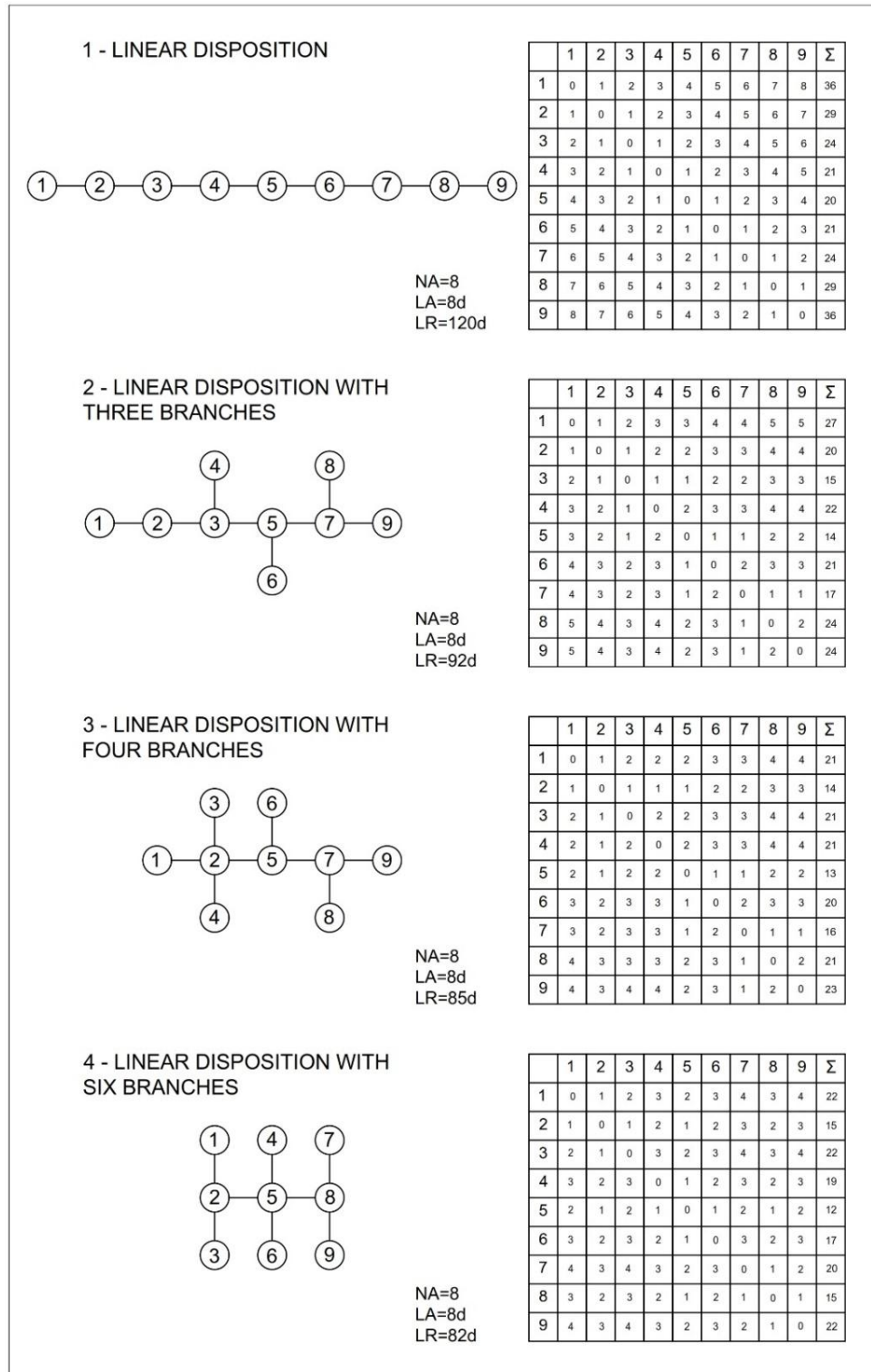
Considering the urban system conceptually as a network of entities, we are able to apply the mathematical tools of systems theory for the analysis of qualitative and quantitative characteristics that indicate the intensity of the relationship between pairs of entities expressed by scalar values. How to use mathematical tools is described in detail by Rafael Boix (2002) in the article *Instrumentos de análisis de redes en economía urbana: caracterización de redes de ciudades mediante el análisis de cuatro estructuras urbanas simuladas*. Using the example of the usual four simulated dispositions and connections of entities of urban systems (monocentric system, hierarchical system in the form of a tree, polycentric system, and fully equipotential reticulate system) it shows the calculation of characteristics (descriptive statistics, cohesion, centrality, influence and power) which enables a numerical synthesis of the characteristics of these complex networks.

Questioning the advantages of different morphologies of connections on the development of entities and the overall network of an urban system is a challenge to which this paper seeks to contribute. Assuming a two-way interaction between entities, the paper examines the influence of the different disposition of the entities within the system and the intensity of the use of connections, which is conditioned by the disposition of the entities and the density of the network.

The purpose of this article is to contribute to previous research on monocentrically, polycentrically and diffusely organized urban systems by operationalizing indicators of their physical characteristics related to the disposition of entities and their connections considered at the fourth level of urban morphology resolution. The aim of the article is, with selected indicators, to quantitatively and qualitatively show the differences in the spatial characteristics of the various dispositions of the entities and the influence of the ways of connection and the density of a network on the frequency of its use.

2. Research Methodology

Figure 1. Linear disposition of urban system entities



Source: Authors' elaboration

Note: NA - number of current system entity connections; LA - total length of current system entity connections; LR - total length of the relative connections of all system entities; d - distance between entities.

Urban systems are characterized by numerous features, so we can distinguish them according to the number and size of entities, their mutual functional connection, spatial disposition and infrastructural connection, i.e. the degree of nodality². Some of the characteristics of either entities or urban systems are determined by quantities that can be read from statistical databases, while other characteristics can be the subject of research in the field of spatial planning.

For the purposes of this work, a research methodology was set up that enables the perception and interpretation of the relationship of one-dimensional indicators of the disposition of entities of urban systems for a certain number of variants of hypothetical dispositions of entities. The research uses the method of quantifying indicators that enable a comparative analysis of the obtained results for selected theoretically possible different forms of disposition of entities, and changes in the number of connections of entities in the urban system.

In this research, the assumptions were made that all entities are of the same size and that the interaction of pairs of entities is egalitarian and two-way. Hypothetical distances expressed by geometric unit sizes and derivatives resulting from different geometries of models of hypothetical dispositions of entities are used as indicators of the mutual relationship of individual entities.

The research considered urban systems with entities distributed in a linear form (Figure 1), in an orthogonal form (Figure 2), and with entities organized in a radial-concentric form (Figure 3).

In order to be able to carry out a comparative analysis of differently distributed entities in the urban system, it was necessary to choose a system with a sufficient number of entities that enables the comparison of all three mentioned models of the urban system. To meet these criteria, a system of nine urban structures was chosen as an example with the smallest number of entities, which enables a comparative analysis of the quantitative aspects of the mentioned models of the disposition of urban system entities³.

For the purposes of simulation and presentation of the selected variants, graphic interpretation in the form of diagrams and matrix of relations is used. For the interpretation of interrelationships between entities, square matrices with the same number of entities are used in all variants of different models of urban systems. The matrices show the analytical-mathematical characteristics of different geometric dispositions. In continuation, the schemes of the selected dispositions of the entities and their connections within the model of urban systems, their matrix of relations and the corresponding sizes of the variables that determine them are presented.

Representative samples of different models of hypothetical dispositions of urban system entities are determined by the following data:

Parameters:

1. $NE = n$ - number of urban system entities
2. $NR = n(n-1)/2$ - number of relative connections of all system entities

Variables:

3. NA - number of current system entity connections
4. LA - total length of current system entity connections
5. LR - total length of the relative connections of all system entities

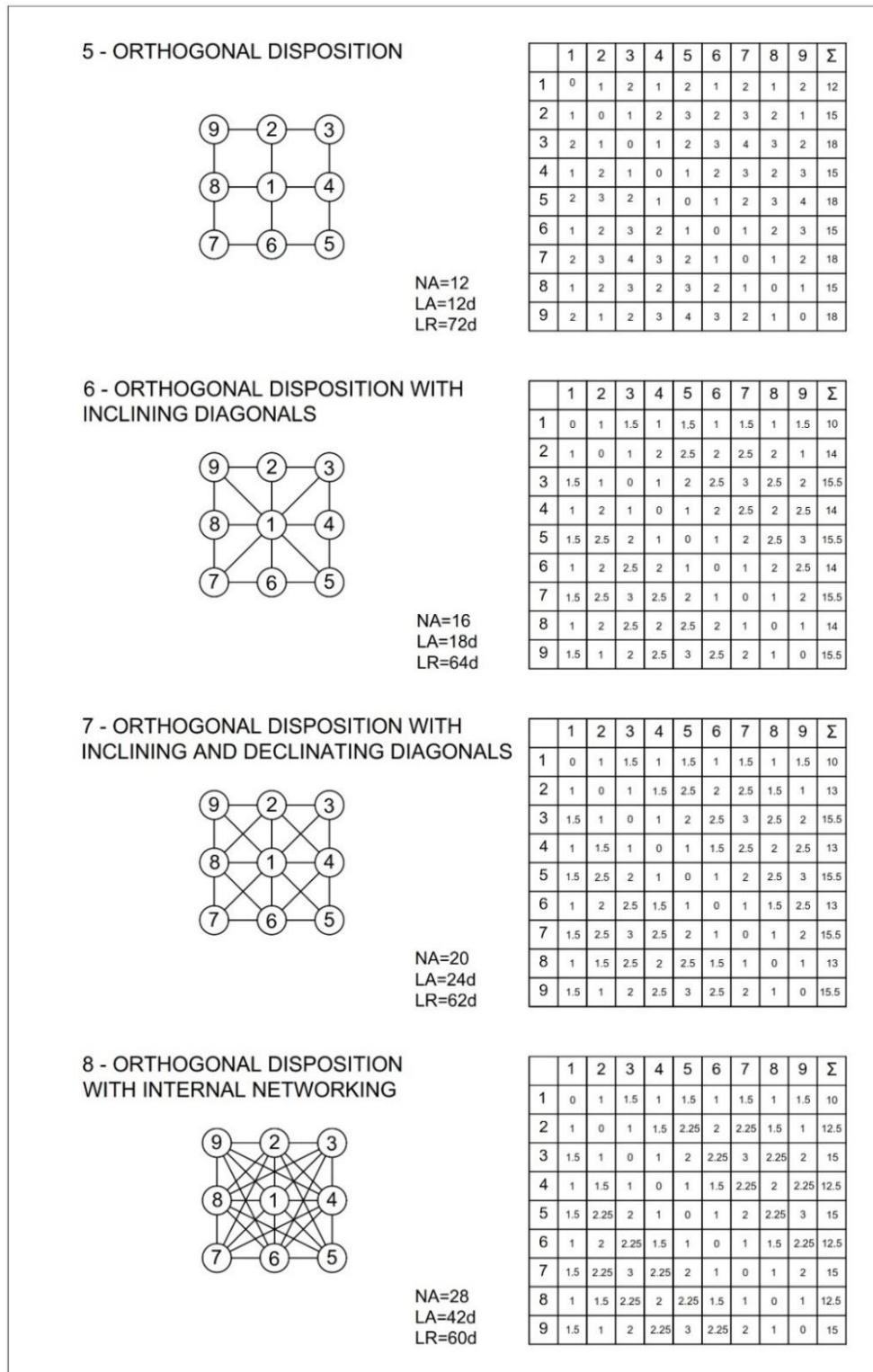
Indicators:

6. CC - coefficient of current connection of entities (NA/NR)
7. CL - coefficient of system connection load (NR/NA)
8. CF - frequency coefficient of the average use of entity connections (LR/LA)

²The degree of nodality is determined by the total number of connections by which the observed entity is connected to other entities of the urban system.

³For all selected dispositions of entities and their connections within the model of urban systems, the following parameters apply; number of entities of the urban system $NE=n=9$; and the number of relative connections between system entities $NR=n(n-1)/2=36$

Figure 2. Orthogonal disposition of urban system entities



Source: Authors' elaboration

Note: NA - number of current system entity connections; LA – total length of current system entity connections; LR – total length of the relative connections of all system entities; d – distance between entities.

The parameters and variables used in the research are one-dimensional, which makes it much easier to analyse the characteristics of variants and models of urban systems. By interrelating parameters and variables, indicators are obtained, namely: coefficient of current connection of entities (CC), coefficient of system connection load (CL) and frequency coefficient of the average use of entity connections (CF). The obtained quantitative values of the indicators make it possible to observe certain quantitative differences between the presented models and certain variants of the model of the disposition of urban systems entities.

In this research, mathematical methods are used, the method of quantifying connections as a result of the spatial connection of entities, and the method of quantifying the interrelationship of parameters (the number of system entities and the number of relative/theoretical connections between system entities) and variables (the number of current system entity connections, total length of current system entity connections and the total length of the relative connections of all system entities), that is, the method of indicator qualification of the different variants characteristics of urban systems models.

For the purposes of interpretation and comparative analysis of all parameters, variables and indicators of certain forms of disposition of the urban system, i.e. their changes in characteristics in different dispositions of entities and morphogenesis of connections, a summary table with all the listed elements was made (Table 1).

To interpret the morphogenesis of connections and the size of indicators for individual models and variants of entity disposition, and since input parameters are of discrete size, diagrams are used that show selected characteristics of urban systems networks. Changes in the number of connections in the urban system are shown on the abscissa, while the values of all three indicators (coefficient of current connection of entities, coefficient of system connection load and frequency coefficient of the average use of entity connections) for the selected variant of the urban system model are shown simultaneously on the ordinate.

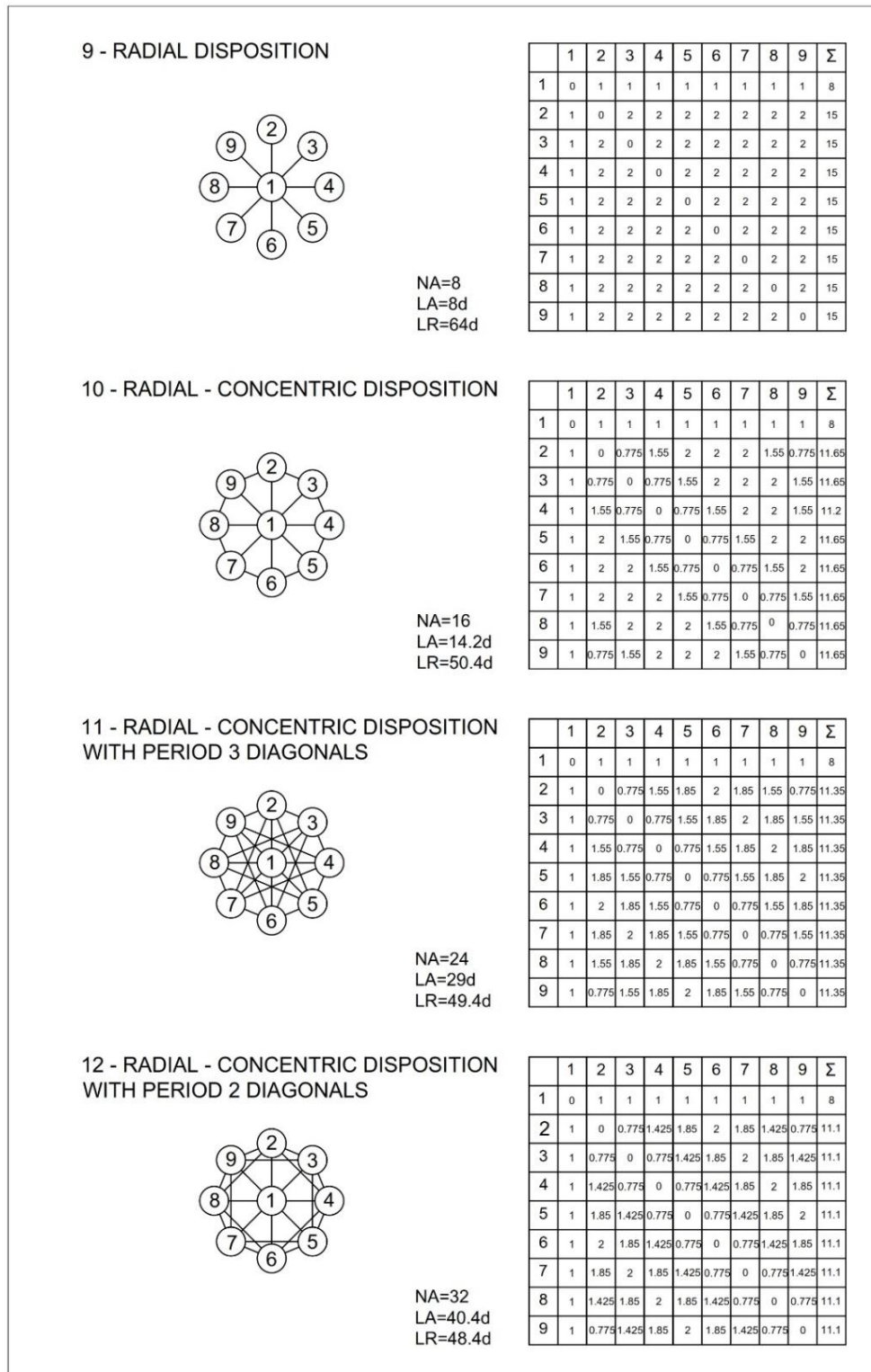
Diagrams are shown separately for the three models and their selected variants (Figure 4, Figure 5 and Figure 6).

For a more detailed look at the changes of reference variables and indicators in certain variants and models of selected urban systems (shown in Table 2) in relation to its base value, there is an analysis of the frequency index of average use of entity connections (I_{xCF}) and the index of path length increase of current system entity connections (I_{xLA}). Furthermore, the values of the mentioned indices are averaged (I_{xCFa} and I_{xLAa}).

In order to see the mutual effect of the path length increase and the frequency of use of the connections of the entities, the average values of the indices (I_{xCFa} and I_{xLAa}) are calculated, which are put into the interrelationship and expressed as the average index of interaction of path length increase and frequency of use of entity connections ($I_{xLACFa} = I_{xLAa} \times I_{xCFa}$) and the average index of path length influence on the frequency of use of entity connections ($I_{xpLACFa} = 1 / I_{xLACFa}$). The analysis results of the observed indices are presented in a table, and their values are commented on in the discussion, putting them in relation with the selected variants and models of the observed urban systems.

The established research methodology is intended primarily for the analysis and interpretation of the characteristics of the fourth level of resolution of urban morphology, i.e. the indicators of the influence of changes in the disposition of entities and the morphogenesis of urban system connections. With certain modifications, the methodology used could also be applied at the third level of resolution of urban morphology, that is, to analyse and interpret the indicators of the relationship between the parts of the composition of the urban structure. Additional modifications of the methodology would enable research into the existing spatial dispositions of entities within an urban system or structure, or for planning new ones.

Figure 3. Radial – concentric disposition of urban system entities



Source: Authors' elaboration

Note: NA - number of current system entity connections; LA – total length of current system entity connections; LR – total length of the relative connections of all system entities; d – distance between entities.

3. Research Results

In the research process, emphasis was placed on the possibility of identifying quantitative and qualitative indicators of the disposition of entities of different models of urban systems networks. Table 1 shows the values of input parameters and variables, as well as the values of indicators for the observed hypothetical dispositions of entities and their connections in individual models of urban systems.

Table 1. Characteristics of Urban Systems Models Determined by Different Dispositions of Entities and the Morphogenesis of Their Connections

| The form of disposition of urban system entities | 1 NE | 2 NR | 3 NA | 4 LA | 5 LR | 6 CC | 7 CL | 8 CF |
|--|---------|------------|---------|------------|----------------|---------|---------|---------|
| | n(v) | n(n-1) / 2 | E | Σd | $5 \Sigma d_r$ | NA/NR | NR/NA | LR/LA |
| 1 Linear disposition | 9 | 36 | 8 | 8d | 120d | 0.222 | 4.50 | 15,00 |
| 2 Linear disposition with three branches | 9 | 36 | 8 | 8d | 92d | 0.222 | 4.50 | 11.50 |
| 3 Linear disposition with four branches | 9 | 36 | 8 | 8d | 85d | 0.222 | 4.50 | 10.62 |
| 4 Linear disposition with six branches | 9 | 36 | 8 | 8d | 82d | 0.222 | 4.50 | 10.25 |
| 5 Orthogonal disposition | 9 | 36 | 12 | 12d | 72d | 0.333 | 3.00 | 6.00 |
| 6 Orthogonal disposition with inclining diagonals | 9 | 36 | 16 | 18d | 64d | 0.444 | 2.25 | 3.55 |
| 7 Orthogonal disposition with inclining and declinating diagonals | 9 | 36 | 20 | 24d | 62d | 0.555 | 1.80 | 2.58 |
| 8 Orthogonal disposition with internal networking | 9 | 36 | 28 | 42d | 60d | 0.777 | 1.28 | 1.42 |
| 9 Radial disposition | 9 | 36 | 8 | 8d | 64d | 0.222 | 4.50 | 8.00 |
| 10 Radial-concentric disposition | 9 | 36 | 16 | 14.2d | 50.4d | 0.444 | 2.25 | 3.54 |
| 11 Radial-concentric disposition with period 3 diagonals | 9 | 36 | 24 | 29d | 49.4d | 0.666 | 1.50 | 1.70 |
| 12 Radial-concentric disposition with period 2 diagonals | 9 | 36 | 32 | 40.4d | 48.4d | 0.888 | 1.125 | 1.19 |

Source: Authors' elaboration

Note: n – number of entities; na – number of current connections; d – unit length of the connection between entities; Σd – the sum of the lengths of current connections between system entities; Σd_r – the sum of the lengths of the relative connections of all system entities; 1 - NE – number of urban system entities (n); 2 - NR – number of relative connections of all system entities (n(n-1)/2); 3 - NA – number of current system entity connections (na); 4 - LA – total length of current system entity connections (Σd); 5 - LR – total length of the relative connections of all system entities (Σd_r); 6 - CC – coefficient of current connection of entities (NA/NR); 7 - CL – coefficient of system connection load (NR/NA); 8 - CF – frequency coefficient of the average use of entity connections (LR/LA)

In the conducted research, the initial parameters, NE - number of urban system entities and NR - number of relative connections of all system entities are identical for all observed models and their variants. Based on the set criteria, the selected total number of entities of the urban systems observed variants (NR) is 9, and the number of relative connections of all system entities is determined by the formula $n(n-1)/2$, and for all twelve variants of the organization of urban systems it is 36.

From the conducted research, it follows that for the model of the linear disposition of the urban system entities, the number of current system entity connections (NA) is the same for all four selected variants, that is, there are 8 connections, regardless of the geometric form of the disposition of the entities in the linear urban system, assuming orthogonal connection of the branches. The total length of current system entity connections (LA) is also identical for all selected variants and is $8d$, while the variable indicating the total length of the relative connections of all system entities (LR) is different for different dispositions, ranging from $82d$ to $120d$, which depends on the geometric disposition of the entities (number of branches) and the lengths of the connections between the entities.

The indicators, the coefficient of current connection of entities (CC) and the coefficient of system connection load (CL), in all variants of the organization of the linear model, show the same characteristics, that is, they have the same values. The coefficient of current connection of entities (CC) and the coefficient of system connection load has a value of 0.222, while the coefficient of system connection load is 4.5. Unlike the previous indicators, the frequency coefficient of the average use of entity connections (CF) is different for all variants of the organization and ranges from 10.25 to 15, as it also depends on the disposition of the entities (number of branches) and the length of connections between entities.

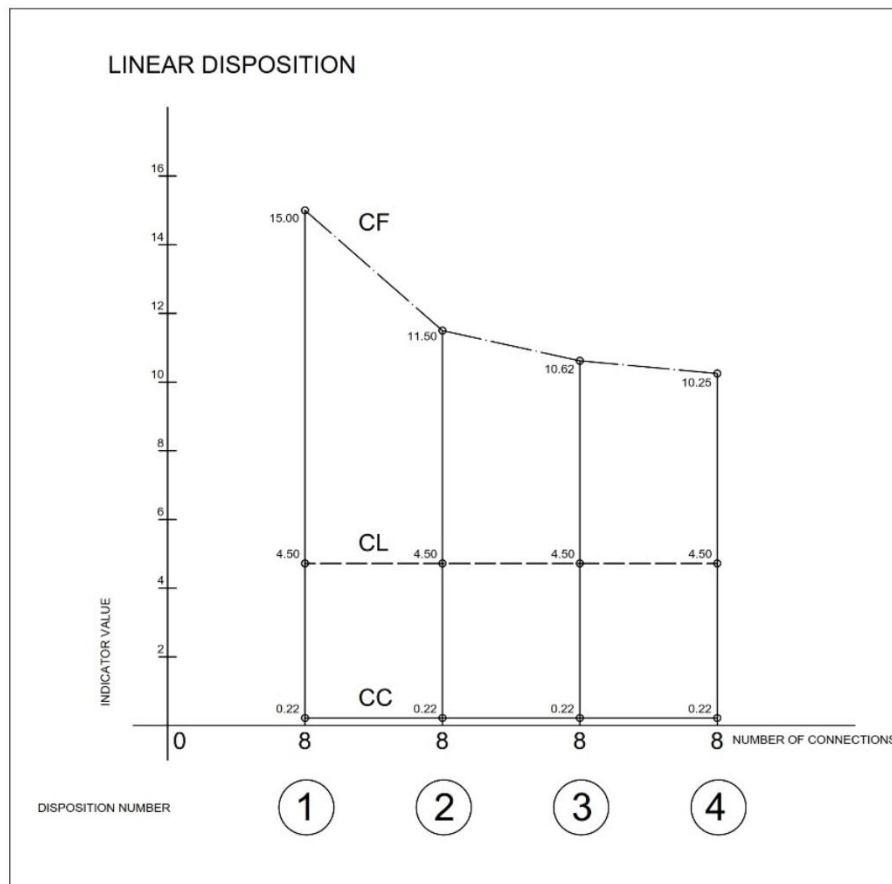
For the four selected variants of the orthogonal disposition model of urban system entities, the variable NA - number of current system entity connections ranges from a minimum of 12 to 28, depending on the degree of entity nodality. The total length of current system entity connections (LA) for the selected variants ranges from $12d$ to $42d$, and the variable indicating the total length of the relative connections of all system entities (LR) is also different for individual disposition variants, ranging from $60d$ to $72d$, which depends on the geometric disposition of the entities (number of branches) and the lengths of the connections between the entities.

Indicators, coefficient of current connection of entities (CC), coefficient of system connection load (CL) and frequency coefficient of the average use of entity connections (CF), in all variants of the organization of the orthogonal disposition model, show different characteristics, that is, their values either increase or decrease depending on the interrelations of individual variables for the observed variants of the model. The coefficient of current connection of entities (CC) ranges from 0.333 to 0.777, the coefficient of system connection load (CL) has a value from 1.28 to 3.00, while the frequency coefficient of the average use of entity connections (CF) ranges from 1.42 to 6.00.

For the four selected variants of the radial-concentric disposition model of urban system entities, the variable NA - the number of current system entity connections ranges from 8 to 32, depending on the degree of entity nodality. The total length of current system entity connections (LA) for the selected variants ranges from $8d$ to $40.4d$, and the variable indicating the total length of the relative connections of all system entities (LR) is also different for individual variants of the disposition, ranging from $48.4d$ to $64d$, which depends on the geometric disposition of the entities (number of branches) and the lengths of the connections between the entities.

Indicators, coefficient of current connection of entities (CC), coefficient of system connection load (CL) and frequency coefficient of the average use of entity connections (CF), in all variants of the organization of the radial-concentric disposition model, show different characteristics, i.e. their values either increase or decrease depending on interrelations of individual variables for the observed variants of the model. The coefficient of current connection of entities (CC) ranges from 0.222 to 0.888, the coefficient of system connection load (CL) has a value from 1.125 to 4.50, while the frequency coefficient of the average use of entity connections (CF) ranges from 1.19 to 8.00.

Figure 4. Comparative display of indicators of linear disposition variants of urban system entities



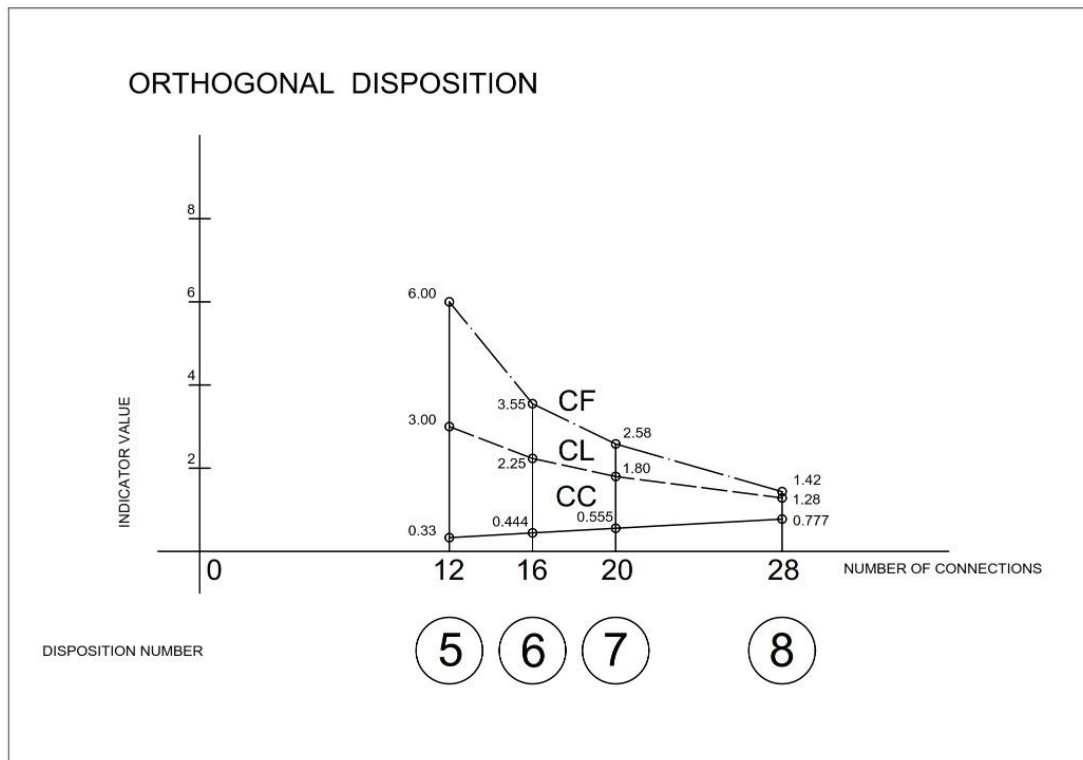
Source: Authors' elaboration

Note: CC – coefficient of current connection of entities (NA/NR); CL – coefficient of system connection load (NR/NA); CF – frequency coefficient of the average use of entity connections (LR/LA)

Based on the results of the research, which are summarized in Table 1 *The characteristics of the model of urban systems determined by the different dispositions of the entities and the morphogenesis of their connections*, diagrams were created that graphically interpret and enable the comparison of the three observed indicators, the coefficient of current connection of entities (CC), the coefficient of system connection load (CL) and the frequency coefficient of the average use of entity connections (CF). Changes in observed indicators for individual models of disposition of urban system entities are shown in Figure 4 *Comparative display of indicators of linear disposition variants of urban system entities*, Figure 5 *Comparative display of indicators of orthogonal disposition variants of urban system entities* and Figure 6 *Comparative display of indicators of radial-concentric disposition variants of urban system entities*. The values of the indicators are based on the morphogenesis of the number of connections, which are mathematically discrete in size.

From Figure 4, which shows the tendencies of the indicators for the linear disposition variants of urban system entities, it can be read that the number of current system entity connections (NA) is always one number less than the number of entities. The coefficient of current connection of entities (CC) and the coefficient of system connection load (CL) has constant values, while the frequency coefficient of the average use of entity connections (CF) decreases where the number of branches of the entity's linear disposition increases. Regardless of the number of system entities, both indicators, the coefficient of current connection of entities (CC) and the coefficient of system connection load (CL) are constant in the case of the linear model.

Figure 5. Comparative display of indicators of orthogonal disposition variants of urban system entities



Source: Authors' elaboration

Note: CC – coefficient of current connection of entities (NA/NR); CL – coefficient of system connection load (NR/NA); CF – frequency coefficient of the average use of entity connections (LR/LA)

From Figure 5, which shows the tendencies of the indicators for the orthogonal disposition variants of the urban system entities, it can be read that the number of current orthogonal system entity connections (NA), for the selected 9 entities, is 12 and increases depending on the selected disposition variants and the number of diagonal connections between the entities.

The values of the coefficient of system connection load (CL) and the frequency coefficient of the average use of entity connections (CF) decrease with the increase in the number of connections between entities. Unlike the previous two indicators, the coefficient of current connection of entities (CC) increases where the number of connections increases.

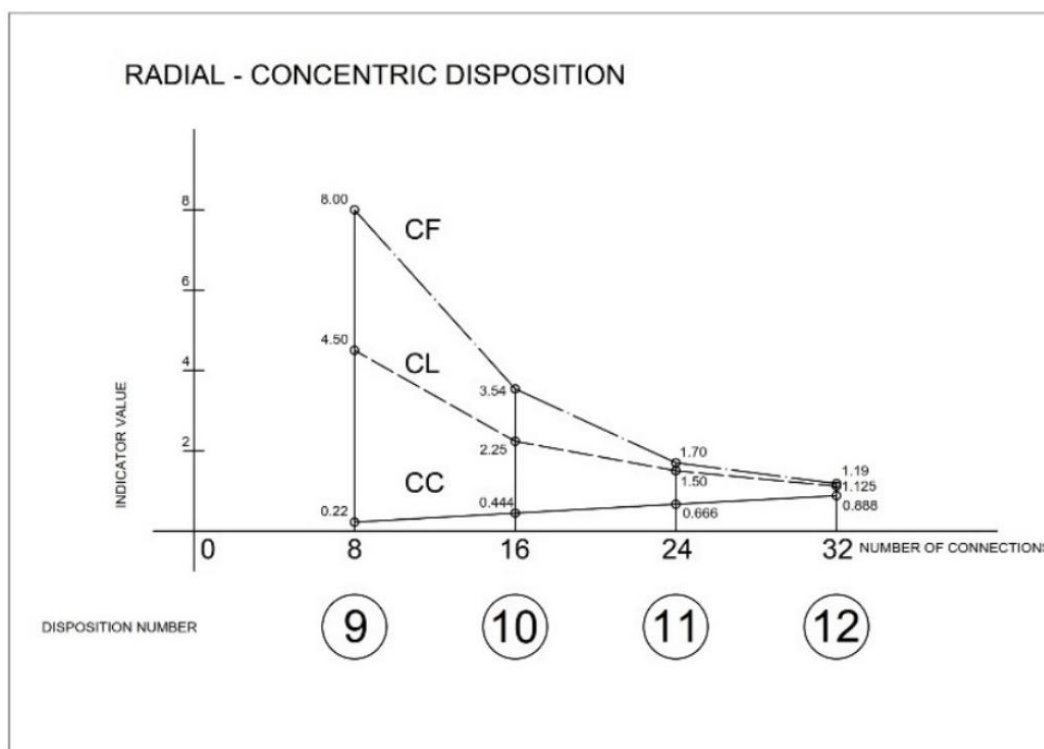
All three indicators take on unit size when the number of actual connections and relative connections of all system entities is equalized.

From Figure 6, which shows the tendencies of the indicators for the radial-concentric disposition variants of the urban system entities, it can be read that the number of current system entity connections (NA), for the selected 9 entities, is 8 and increases depending on the morphogenesis of the connections, that is, the model variant observed.

The values of the coefficient of system connection load (CL) and the frequency coefficient of the average use of entity connections (CF) decrease where the number of connections between entities increases. Unlike the previous two indicators, the coefficient of current connection of entities (CC) increases where the number of connections increase.

All three indicators take on unit size when the number of actual connections and relative connections of all system entities is equalized.

Figure 6. Comparative display of indicators of radial-concentric disposition variants of urban system entities



Source: Authors' elaboration

Note: CC – coefficient of current connection of entities (NA/NR); CL – coefficient of system connection load (NR/NA); CF – frequency coefficient of the average use of entity connections (LR/LA)

4. Discussion

From the presented schemes of different model variants and diagrams, it is evident that with the linear model, the number of current system entity connections (NA) is always one less than the number of entities, regardless of the observed system variant. In orthogonal and radial-concentric models, the number of current system entity connections (NA) increases depending on the observed variant, in the orthogonal model depending on the diagonal connections of the entities, and in the radial-concentric model depending on the measured period of connection of urban system entities.

The coefficient of current connection of entities (CC) and coefficient of system connection load (CL) are constant in all observed variants of the linear model, regardless of the number of system entities. In contrast to the linear model, in the variants of the orthogonal and radial-concentric models, the coefficient of current connection of entities (CC) increases where the number of connections increases, while the coefficient of system connection load (CL) decreases where the number of connections in the observed variants of urban networks increase. From the conducted research, it can be seen that in all variants of the three observed models, the frequency coefficient of the average use of entity connections (CF) decreases. In the case of the linear model, this is the result of an increase in the number of branches, and in the case of the other two models, it is a consequence of the increase in the number of connections, i.e. the total length of current system entity connections (LA).

All three indicators (CC, CL and CF), in the case of orthogonal and radial-concentric models, assume a unit value when equalizing the number of current (NA) and relative connections (NR) of the network of system entities. In the case of a linear model, these indicators converge asymptotically with an infinite number of system entities.

From Figure 5 and Figure 6, it can be read that the coefficient of current connection of entities (CC) and the coefficient of system connection load (CL) in the orthogonal and radially concentric model variants would have the same values for the same number of entity network connections, because they depend on the ratio of the number relative and current connections of system entities.

Table 2 Indices of Relations of Urban Systems Models Determined by Different Dispositions of Entities and the Morphogenesis of Their Connections

| Models | Linear | | | | Orthogonal | | | | Radial-concentric | | | |
|-----------------------------------|---|-------|-------|-------|---|-------|-------|-------|---|--------|--------|--------|
| Variants | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| Indices | x1/x1 | x2/x1 | x3/x1 | x4/x1 | x5/x5 | x6/x5 | x7/x5 | x8/x5 | x9/x9 | x10/x9 | x11/x9 | x12/x9 |
| I _x CF | 1.00 | 0.76 | 0.70 | 0.68 | 1.00 | 0.59 | 0.43 | 0.23 | 1.00 | 0.44 | 0.21 | 0.14 |
| I _x CF _a | 0.71 | | | | 0.41 | | | | 0.26 | | | |
| | x1/x1 | x2/x1 | x3/x1 | x4/x1 | x5/x5 | x6/x5 | x7/x5 | x8/x5 | x9/x9 | x10/x9 | x11/x9 | x12/x9 |
| I _x LA | 1.00 | 1.00 | 1.00 | 1.00 | 1.00 | 1.50 | 2.00 | 3.50 | 1.00 | 1.77 | 3.66 | 5.05 |
| I _x LA _a | 1.00 | | | | 2.33 | | | | 3.49 | | | |
| | I _x CF _a x I _x LA _a | | | | I _x CF _a x I _x LA _a | | | | I _x CF _a x I _x LA _a | | | |
| I _x LACF _a | 0.71 | | | | 0.96 | | | | 0.91 | | | |
| | (1 / I _x LACF _a) | | | | (1 / I _x LACF _a) | | | | (1 / I _x LACF _a) | | | |
| I _x pLACF _a | 1.40 | | | | 1.04 | | | | 1.09 | | | |

Source: Authors' elaboration

Note: x1-12 – the value of the variables and indicators from Table 1, depending on the variants of the entities disposition; I_xCF – Frequency index of average use of entity connections; I_xCF_a – the average value of the frequency index of average use of entity connections; I_xLA – Index of path length increase of current system entity connections; I_xLA_a – the average value of the index of path length increase of current system entity connections; I_xLACF_a – average index of interaction of path length increase and frequency of use of entity connections (I_xLA_a x I_xCF_a); I_xpLACF_a – average index of path length influence on the frequency of use of entity connections (1 / I_xLACF_a)

From Table 2 Indices of relations of urban systems models determined by different dispositions of entities and the morphogenesis of their connections, it is clear that for the linear model, regardless of the constant value of the index of path length increase of current system entity connections (I_xLA=1.00) in all observed model variants, the frequency index of average use of entity connections (I_xCF) changes. By increasing the number of branches of the observed variants of the entity disposition model, the index I_xCF decreases, and its average value of the considered variants is 0.71 (I_xCF_a).

The average value of the frequency index of average use of entity connections (I_xCF_a) for the orthogonal model is lower than the same index of the linear model, and is 0.41. Table 2 shows a significant increase in the average value of the average value of the index of path length increase of current system entity connections (I_xLA_a), compared to the same index of the linear model, which amounts to 2.33.

The average value of the index of path length increase of current system entity connections (I_xLA_a) of the radially concentric model of the disposition of entities increases significantly compared to the previous models and amounts to 3.49, and the average value of the average value of the frequency index of average use of entity connections (I_xCF_a) decreases to the amount of 0.26.

From the obtained results, it follows that average index of interaction of path length increase and frequency of use of entity connections (IxLACFa) for the linear model is 0.71, for the orthogonal model 0.96, and for the radial-concentric model 0.91.

The average index of path length influence on the frequency of use of entity connections (IxpLACFa) differs significantly in the comparison of the sizes of the linear model and the other two considered models. In the case of the linear model, there is considerable disproportionality, which is a consequence of the interpolation of the branches, while the disproportionality in the other two models is small (1.04 in the orthogonal and 1.09 in the radial-concentric model).

5. Conclusion

The research conducted yielded indicator values and their tendencies of influence on changes in the dispositions and morphogenesis of connections between entities of urban systems in the linear, orthogonal and radial-concentric models and their variants depending on the spatial disposition and interconnections of the entities. Based on the research, it can be concluded that changes in parameters and variables can significantly affect changes in the values of indicators and the average values of the urban system indices.

By increasing the number of network branches and current connections in the network, the characteristics of the urban system model change, which is evident from the quantitative changes in the values of the analysed indicators - the coefficient of current connection of entities (CC), the coefficient of system connection load (CL) and the frequency coefficient of the average use of entity connections (CF).

Considering and comparing all three models, we can conclude that the average value of the frequency index of average use of entity connections (IxCFa) is the highest in the variants of the linear model, and the lowest in the radial-concentric model. Furthermore, we can conclude that the inverted features show the average value of the index of path length increase of current system entity connections (IxLAa), which has the highest value in the case of the radial-concentric model, and the lowest in the case of the linear model. The average index of interaction of path length increase and frequency of use of entity connections (IxLACFa) in the orthogonal model has a higher value than in the radial-concentric model due to the geometry of the system network formed by the external links of the entities, while the average index of path length influence on the frequency of use of entity connections (IxpLACFa) has the highest value in the case of the linear model due to the reduction in the frequency of the use of connections by branch interpolation.

The specificities of the researched models and the tendencies of indicator values indicate the possibilities of further research into the relationship between the disposition and morphogenesis of connections between urban system entities in the context of analysing other characteristics such as accessibility, transitivity, association, hierarchy, etc., and research related to the creation of hybrid models of urban systems.

The applied research methodology and interpretation of the obtained results show the possibility of exact calculation of indicators of quantitative characteristics and interpretation of qualitative characteristics of a certain form of disposition of the urban system entities for the purpose of correcting the existing disposition or generating a new disposition model at the fourth level of urban morphology resolution regardless of the number of entities of the urban system. The applied methodology, with certain modifications, can also be used for the analysis of compositions of urban structures at the third level of urban morphology resolution.

The obtained results can also serve as a basis for further research into the characteristics of the spatial component of socio-spatial constructs. Whether it is an urban or rural environment, in addition to other elements of spatial formations of natural and artificial origin, further research should also include processes, as consequences of the daily functions of individual entities and urban systems.

By further researching the interrelationships and values of indicators of other components (economic, social, ecological) of different models, more complete evidence can be obtained that indicates the advantages of certain models of the disposition of entities in urban systems, which is considered relevant for further study of sustainability in urbanism and spatial planning. For this purpose, mathematical operators should be used for research, both for the quantification and qualification of static (spatial) features and for dynamic (social) features in space. The proposed approach would contribute to the clarification of dilemmas about the differences in the efficiency of individual models of urban systems in space, and potentially reduce conflicting differences in the reflections of scientists on the use of individual models of urban systems in the context of sustainability and sustainable development.

Acknowledgements

The article is part of the research project *Challenges of contemporary cities and regions*, financed by the University of Zagreb, Faculty of Architecture, led by Associate Professor Lea Petrović Krajnik. We thank the Editorial Board and reviewers for their comments and support in this research.

Authorship

The first and second author have conceptualized and designed the research, analysed the data and written the work; the third author has analysed the data and made graphic representation of the work.

Conflict of interests: The authors declare that there is no conflict of interest.

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