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REVISITING EMPLOYMENT DENSITY AS A MEANS TO DETECT METROPOLITAN SUB-CENTRES: AN ANALYSIS FOR BARCELONA AND MADRID

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Abstract

Polycentrism is seen as a desirable model of metropolitan structure. The starting point in its study rests upon the conceptualisation and identification of sub-centres. The hegemonic family of sub-centre identification is based on the analysis of density. Huge efforts have been devoted to achieving statistically robust models, but little attention has been paid to the conceptualisation of density itself. This paper presents a different approach to calculating density through the introduction of basic elements of mobility and placing priority on sub-centres as those municipalities that are closer to the urban paradigm of central-cities in southern-Europe —they are attractive both for living and working. The analysis in Barcelona and Madrid suggests that compound-density allows for selecting what local experts would define as a sub-centre, discarding most dormitory, adjacent-to-CBDs and small municipalities, found by classical-density. Furthermore compound density sub-centres are denser, they contain more central-activities and are strongly linked to other municipalities.

1. Introduction

Changes in metropolitan areas characterised by the dispersion and concentrated decentralisation of employment and population, have led to a specialised line of research into polycentric urban systems (Dematteis, 1998; Anas et al., 1998; Kloosterman and Musterd, 2001b; Cattán, 2007). The topic is of obvious interest because a perfectly polycentric system

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would offer the two major economic advantages of urban systems: the presence of agglomeration economies, which result in increasing returns for companies, and a potential reduction of transport costs (including time), which lead to a reduction in salaries and land rent (McMillen and Smith, 2003; McMillen, 2003c; McDonald, 2009). Moreover, this urban model would have social and environmental benefits arising from an improvement in transport planning (McMillen, 2001b) and a drop in commuting (Gordon et al. 1986) if the network were designed to connect sub-centres (McMillen, op. cit.). In theory, polycentric systems offer the benefits of large and medium-sized cities (McMillen and Smith, op. cit.) by combining the advantages of traditional centralised cities with a decentralised spatial configuration (McMillen, 2003b). Moreover, Champion (2001) has envisaged the existence of a relationship between this form on spatial arrangement and the so called second demographic transition (i.e. ageing, non-family households, and the rise of ethnic communities), in which polycentrism would facilitate the daily dynamics of such emergent demographic model.

The European Spatial Development Perspective (ESDP), agreed in 1999, proposes the promotion of polycentrism as a European Union central policy. At a continental scale polycentrism is understood as the promotion of alternative centres, outside the so-called *pentagon*, but at a regional level, the polycentric notion is associated with a concentrated-decentralisation from central cities to emergent ones functionally linked amongst themselves, but not necessarily contiguous to one another.

Nevertheless in studying the polycentrism of metropolitan regions in order to promote it, or analyse its efficiency, one crucial issue remains to be solved i.e. how to measure the level of polycentricism. In this respect some authors (Gordon and Richardson, 1998; Giuliano and Redfean, 2005) have suggested the percentage of employment located in sub-centres in relation to that spread out throughout a wider territory as an indicator of polycentrism. Others have suggested the level of influence of sub-centres on the structure of their surroundings (McMillen, 2001a; Muñiz et al., 2003). So the starting problem, in the measure of polycentrism, is the detection of sub-centres. This paper seeks to make an advance in this field through conceptualising the notion of a sub-centre and finding indicators capable of detecting them.

According to McMillen “a reasonable working definition of a sub-centre is a location⁴ with significantly larger employment density than nearby locations that has (2) a significant effect on the overall employment density function” (McMillen, 2001: 448-449). While this latter conceptualisation derives directly from the *standard urban model* (i.e. that based on the bid rent theory) and performs well when detecting dense employment zones departing from relatively small areas such as census tracts (or even districts within a city), it does not suffice, when what is pursued is the identification of more complex centralities or mature subcentres, some of the resulting from the integration of formerly independent nucleus.

In the literature huge efforts have been devoted to achieving statistically robust models of employment density, but very little attention has been paid to the conceptualisation of density. The simply notion of a statistically dense location does not guarantee that the previously stated

⁴ Defined by the metropolises of London, Paris, Milan, Munich and Hamburg. In this respect the ESPON 1.1.1 and ESPON 1.4.3 projects have tried to create a quantitative measurement of the extent of policentricity based fundamentally in a morphological (i.e. size and spatial distribution of FURs inside UE countries) approach Meijers (2008).

metropolitan sub-centre conditions are met. It is necessary, therefore, to revisit the employment density to see if it is possible to make a distinction between simple employment concentrations and more complex centralities or mature subcentres.

The aim of this study is to advance towards an alternative means of calculating employment density by introducing elements of metropolitan mobility in the context of the identification of metropolitan sub-centres. Of course, we do not intend to give a definitive solution, but mainly to centre the researchers attention on the fact that employment concentration is not the only attribute of central cities, nor should be of alternative to CBD centres.

The remainder of the paper is organised as follows: i) first the theory on density formation and derived empirical methods to detect sub-centres are discussed; ii) a critical analysis of employment density used by the aforementioned methods in the framework of what in the context of this paper is considered to be a metropolitan sub-centre is presented; iii) some proposals for computing compound density are devised and by discarding some of them one is given priority; iv) the efficiency of prioritised compound density is tested in the two largest (but structurally different) Spanish metropolises; and v) the results are placed in perspective in the final remarks.

2. Density formation and methods used to identify intrametropolitan sub-centres in literature

2.1 Theory subjacent in density formation

For urban economists density is an essential feature of urban economy, since frequent contact between different economic activities is feasible only if firms and households are concentrated in a relatively small area (O'Sullivan, 2007).

The standard urban model as shaped by Alonso (1964), Muth (1969) with roots in the pioneering work of Von Thünen (1826) and Laundhardt (1885) is the theoretical framework behind the formation of urban densities. This model, originally conceived for a monocentric city, explains that in achieving locational equilibrium households bid for land according to costs saved in commuting. Thus the closer the place of residence to the CBD (where all employment is supposed to be located) the higher the rent transferred to land (which capitalises into higher prices), resulting in a land rent gradient. It is the existence of land rent gradients that underlie the formation of density in a competitive market scenario. If it is considered that house builders invest capital in land and building when developing a site, and constant returns per unit of land are relaxed (i.e. once substitution between land and building cost is allowed), house builders economise on the use of land in more central locations where prices peak. In optimising developments they add more building capital per unit of capital invested on land, i.e. they build multi-storey structures instead of low rise ones, resulting in a density gradient following that of land prices. Of course the parallelism between rent and density gradients depends upon the elasticity of substitution between land and capital (Kau and Lee, 1976). In this way the willingness to pay for job-accessible locations is the mechanism lying behind land rent and density gradients.

Mills and Hamilton (1984) demonstrated, starting from the monocentric city model, that under certain constraints, such as Cobb-Douglas's production function for housing, users with identical tastes and income and unit price elasticity of demand for housing, density gradients adopt a negative exponential function. According to McDonald (1987) excellent review it was Stewart (1947) who apparently first empirically used the negative exponential function to test whether or not population densities decrease with the distance to the CBD. McDonald notes that it was Clark (1951) who popularised this amongst scholars. Bertaud and Malpezzi (2003) found that such a functional expression fits relatively well in several cities around the globe.

When the premise that centres are only employment concentrations is relaxed, as seems to be taken for granted in the monocentric model conceived in the US in the time of exclusive zoning, it is expected that other (central) attributes like service and amenities may also generate externalities with incidence on the spatial formation of land values and densities as will be argued further.

If the monocentric city model is reformulated by introducing the existence not only of agglomeration economies which induces employment to concentrate in one location, but also the existence of diseconomies of agglomeration (e.g. congestion) which induces employment to decentralise, it is possible to obtain a polycentric city model (White, 1999). Of course polycentrism can be achieved not only by the decentralisation of employment from the CBD due to the existence of centrifugal forces, but also by the functional integration of formerly independent urban centres. This latter line is affiliated to Central Place Theory (one of the first in recognise polycentrism in a interurban scale) which considers that market areas are defined by individuals' willingness to travel for achieving the consumption of centrally distributed goods and services (Christaller, 1933). In this respect when travel costs are reduced the expansion of market areas allows for the integration of central places as sub-centres (Champion, 2001). Whether polycentric urban structures come from decentralisation or integration, the continuing argument in urban economic theory is that both overall land rents and density gradients are conjointly influenced by the proximity to the CBD and sub-centres. Sub-centres, therefore, mimic at the local scale the influence that is exerted by CBD at the global scale.

The vast majority of methodologies have focused on the identification of sub-centres by studying either a) the density in employment terms of a location (controlling or not the distance to the CBD), or b) the influence of a location in organising the commuting flows in a more complex urban system. Such criteria have clearly defined two families of sub-centre identification.

2.2 *Methods based on the analysis of density*

The first morphological family, based on the analysis of density, is by far the most widespread and widely developed in US since the 80's. This family has four major methodologies:

- 1) The first criterion suggested by McDonald (1987) is based on the identification of employment density *peaks* (the author suggests that a sub-centre is the second peak beyond the CBD). This criterion consists of analyzing density employment to detect local disruptions with the aid of a geographic information system (GIS). Alternatively, the employment/population

ratio can be used to detect the areas that have higher relative concentrations of economic activity. Gordon et al. (1986) restricted the number of sub-centres to those areas with high *t*-values; this line of research was continued by McDonald and McMillen (1990) and Craig and Ng (2001).

2) The second approach consists of using upper and lower cut-offs. This line was originally proposed by Giuliano and Small (1991), who considered sub-centres to be the contiguous census tracts with a density of more than 10 employees per acre and a total critical mass of at least 10.000 jobs. Therefore sub-centres must to meet density and critical mass criteria. The references of this method are Song (1994), Cervero and Wu (1997), McMillen and McDonald (1997), Bogart and Ferry (1999), Anderson and Bogart (2001), Shearmur and Coffey (2002) and Giuliano and Readfean (2007). In this line, García-López (2007 and 2008) and Muñiz and García-López (2009 and 2010) suggested that sub-centres are zones with a density higher than the metropolitan average and at least 1% of metropolitan employment. Hall and Pain (2006) have defined *cores* in their Interreg IIB Polynet Project, as NUTs 5 with 7 or more workers per hectare, and at least 20.000 workers in either single.

3) From an econometric perspective, there is a third methodology that identifies potential sub-centres by analysing significant residuals in an exponential negative density function discussed in 2.1. McDonald and Prather (1994) suggested several models for detecting sub-centres based on the identification of areas with positive residuals that are significant at a 95% confidence level.

4) The fourth approximation (derived from that presented in 3) is based on non-parametric models (e.g., locally or geographically weighted regression –L or GWR-) to detect *peaks* that locally adjust the density function and prioritise the effect of neighbouring municipalities on the adjustment process (McMillen, 2001b; Craig and Ng, 2001; Readfean, 2007). The main advantage of this method is that it enables local gradients of density reduction to be determined across the metropolitan area. Suarez and Delgado (2009) develop a hybrid method, where once that peaks of density have been detected by means of GWR residuals, adjacent census tracks are added to comply with a threshold number of workers and density.

According to McMillen (2001b) approaches based on cut-offs are useful because they enable an historical analysis of the sub-centre structure. Nevertheless, they rely excessively on local knowledge to calibrate the thresholds of critical mass and density, and this can be a problem when trying to compare different metro areas with different local experts. The work of García-López (2007) seems to make an advance by relating the mass cut-off to 1% of metropolitan employment and minimum density to the metropolitan average. Nonetheless, such a criterion, as proposed by García-López, is flawed since the larger the number of spatial units in the metro area, the higher the difficulty to reach the critical mass criterion, and the more homogeneous the density function across units, the higher the probability that a large number of units be above average density. Additionally, the cut-offs approach has a more serious defect in that it tends to prioritise central areas as sub-centres, since they fail to take into consideration what is essential in the standard urban model (i.e. global density is determined by proximity to the CBD). Some authors have tried to solve such a problem by manually removing what they consider to be the CBD, while others have established differentiated thresholds in relation to centrality.

Econometric models have made a significant advance, in conceptual terms, by controlling the influence on overall density exerted by the CBD, approaching in this way the central theory behind density formation. From this perspective sub-centres are locations where the density is significantly higher than that explained by their proximity to the CBD. Therefore, one part of their density is endogenously explained, and this comes into play in differentiating them from other locations. Nonetheless almost all of the econometric methods have failed by constraining the complexity of metropolitan areas to just one single dimension - the distance to the CBD. Notably the density function is affected by specificities lying in three dimensions. Some studies have broken down this limitation by analysing metropolitan corridors; however the results of such analyses are difficult to be conjointly interpreted. Advances in spatial modelling have solved such an issue by explicitly introducing the effect of bi-dimensional space, as in the locally or weighted non-parametric models.

While these methodologies have made a significant advance in understanding the structure of contemporary metropolises, all of them have failed to conceptualise what really lies behind employment density. Departing from employment density as it has been calculated in these studies there are no guarantees that *dense locations* are the random result of urban development or whether they respond to complex metropolitan sub-centres.

2.3 Methods based on the analysis of functional relations

The second family of methods is based on the understanding that sub-centres are not only abnormally dense zones in the metropolitan space, but also structural nodes that can strengthen the functional relationship with their surrounding areas. In that sense, this approach is closer to the conception that centres in a network of cities function as nodes, without the necessity of being dense locations. The methods based on the analysis of functional interactions were designed to delimit territorial systems (Nel-lo, 1998), including Travel To Work Areas in England, Statistical Metropolitan Areas in the USA and Functional Urban Areas, and some focused on detecting sub-centres that structure such territorial systems. References in this field include Bourne (1989), Gordon and Richardson (1996), Burns et al. (2001), Roca and Moix (2005) and Roca et al. (2009).

According to Roca et al. (2009), by analysing the interaction among municipalities in a metropolitan system using the interaction value originally used by Roca and Moix (2005) and inspired in the pioneering work of Smart (1974), the most interlinked municipalities can form a proto-system. The bigger municipality in employment and density terms, within a proto-system, is also the one that has the most intense functional relationships with remaining municipalities inside of proto-system. The method proposed is a bottom-up procedure: first the influence area is delimited and then the point with the maximum interaction is detected (i.e. the centre that attracts and issues proportionally more residence-work flows).

Up to now the methodologies exposed above seem to take alternative and not communicated ways to detect sub-centres, relegating the fact that, when the scale of analysis of polycentrism is a FUR, density and mobility are two faces of the same coin, and therefore, can be integrated in one composed density.

3. Revisiting employment density as a means of detecting metropolitan sub-centres

3.1 *In search of the conditions that metropolitan subcentres should meet*

The search of subcentres necessarily requires a previous definition of them. In the vast majority of literature surveyed the concentration of employment is the only requirement that sites must meet in order to be considered subcentres.

“Employment subcentres are defined as areas with significantly higher employment density than neighbouring sites” (McMillen, 2003b: 57).

Sometimes beyond employment density, a minimal critical mass is asked, as well as some degree of sectorial specialization:

“Employment centers are identified as places that exceed a threshold employment density and a threshold employment level. They are also characterized as specializing on the basis of location quotient analysis” (Anderson and Bogart, 2001: 148).

Although the notion that must exert any influence on their surrounding territory is also present, since it is central on the standard theory:

“An employment subcenter is a concentration of firms large enough to have significant effects on the overall spatial distribution of population, employment, and land prices” (McMillen, 2003a: 3).

And these definitions, that mimic the CBD concept, are basically right because they refer to employment subcentres and not to a more complex idea of centrality. May be for that reason when subcentres are searched in Spanish metropolises analysing only the employment density at municipal level, emerge what local experts would say is a (sub) central municipality, but mixed with small irrelevant sites, suburban anodynes business and industrial parks and even next-to-CBD-dormitory cities (that are dense because of the low standards of their urbanizations built during the dictatorial era), sites these three latters that no one would say are centralities and definitely do not represent any important point in the daily life of citizens, except, of course for the commuters that work in.

Centralities as places are sites that beyond being employment and service concentrations (their main feature) are distinctive zones able to give identity to surrounding territory, not only because their historic preservation (when present), but also by the existence of cultural and advanced urban amenities usually present in mixed used areas, where public space plays an important role, and normally have an active public government. And certainly such features exist beyond traditional city centres, have documented for the case of Japanese suburban technopoles in which not only employment and housing gather together but also other amenities

able to give identity to such places, in a scheme linked to the Howard's garden city idea. Moreover the concept of Edge City proposed by Garreau (1991) might be closer to the idea of such centralities: a non-central area that, despite having more jobs than residents, is economically diversified because it combines qualified office activities with consumption centres where specialised and cultural goods and services are offered (e.g. art galleries). An idea that previously had been conceived by Cervero (1989), when looking for *activity centres* characterised by: density of employment, net inflow of morning commuters and heterogeneity of land uses.

Furthermore some edge cities tend to gain complexity: "alongside the growing economic gravity of post-suburbia, we can consider whether such settlements have become more fully urban in other respects. Arguably some of the new settlements have begun to acquire governmental functions and civic spaces alongside the greater density and mixed use of development and buildings" (Phelps and Wood, 2011: 2.595). And exist not only in suburban campus-like development areas, also in complex metropolitan suburban agglomerations like those found by Gilli (2009) in Paris, where subcentres (including *villes nouvelles*) tends to the employment diversification because of the importance of domestic and endogenous activities (some of them fostered by local based population).

And certainly do exist when intraurban polycentrism derive from the fusion of ancient independent cities as in southern Europe, centralities with a well-defined identity, civic arenas and political functions. Such integration might occur because of the expansion of the original influence areas or the decentralization of employment and population from the dominant centre. As Bontje and Burdack (2005) have pointed out "the higher population densities and denser urban network mean that medium size towns in Western Europe often act as the focus around new functional centres in the outer hinterland of the metropolises crystallise" (Bontje and Burdack, 2005: 319). In this latter respect European edge cities would resemble better as in the edge of the city. Even in Europe, new planned economic suburban poles result from a much higher level of involvement of the public sector which capitalises in more balanced schemes able to accept a wide diversity of land uses and households types.

Not surprisingly one of the common features of above stated examples of non-CBD centralities is the existence of population and not only employment. It is especially relevant when spatial units of analysis are considerably bigger than census tracts or their groupings as is the case in the greater part of the empirical studies referred to in Section 2. In the context of the so called second demographic transition population is symptom of the presence of amenities and site attractiveness "affluent workers are finding the city not simply a clear destination for work but also to live and play" (Clark et al., 2002: 500).

According to Silvestro and Roca (2007) having local population allows (although not always) for the generation of cultural services and for the construction of an identity. Both elements might reinforce the externalities exerted by employment concentration on the surrounding territory, in terms of functional interactions and consequently in land rent and density formation. Having population also allows for bidirectional linkages, which become an important feature in mature subcentres allowing for the knowledge exchange (Boix and Trullén, 2012) and the complementarity of labour-housing markets. So far McMillen (1996) stated that Chicago's monocentric pattern already ceased to exist in the 1960s in front of the emergence of

subcentres understood as edge cities with multifunctional concentrations of office, retail, leisure and housing areas (McMillen and McDonald, 1997). So it is possible to say that such centralities, identified in this paper as metropolitan subcentres, are: a) sufficiently attractive in residential terms to have a resident population; b) sufficiently attractive in employment terms to retain part of their working population; and c) sufficiently diverse to attract workers from elsewhere.

The question remains to what extent one can use such elegant approach, as that based on density analysis, to put apart simple employment clusters from more complex centralities.

3.2 Another way to see employment density

Criticism on employment density, as it has been conceived and calculated in literature, needs to be focused on its *aggregated treatment*. This simplification makes it impossible to distinguish between density generated by commuters (incoming flows) and density that is endogenously generated, i.e. by the population who works in their place of residence (resident workers). This means that classical or *aggregated* density cannot be used to differentiate municipalities that are dense because they attract commuters and simultaneously retain resident workers (mature sub-centres with implications on the territorial structure) from those that are *accidentally* dense and lack major attraction flows (employment peaks without functional relationships with neighbouring municipalities) and from those that are dense but are unable to have a resident population or to retain resident workers, as discussed below. In the literature, employment density is calculated by dividing the total employment by the net urban area⁵, which without doubt leads to the loss of valuable information.

According to Aguirre and Marmolejo (2009) an alternative means of computing density is by carrying out a separate analysis of the density of commuters from other municipalities (incoming flows, or IF) and the density of resident workers (RW). The sum of IF and RW gives the total employment - the larger the IF density of a given municipality, the higher its capacity to attract workers; the larger the RW density the higher the capacity of a municipality to retain its employed residents⁶. If outgoing flows of density - OF - (working population employed in other municipalities) are also considered, it is possible to detect municipalities unable to retain their employed residents. From a conceptual framework, three types of employment concentrations can be defined:

Type 1: Municipalities whose density basically stems from IF. They either specialise in economic activity or have a small or no resident population (e.g. industrial parks in Barcelona or transnational *headquarter cities* in Madrid), or they have a labour force that does not match the profile of the area's labour demand, so OF are considerable. It is difficult to consider them complex centralities.

Type 2: Municipalities whose density mainly originates from RW. They are those with few or no functional links to their environment, i.e. those that are autonomous in labour market terms (e.g.

⁵ Without distinguishing the land effectively used by firms.

⁶ Please note that RW density and OF density are not exactly opposites. Although in this case they respectively represent the capacity and incapacity of a municipality to retain its working population.

manufacturing colonies in the past or military headquarters in the present). Again, it is not possible to consider this kind of area a centrality without spatial interactions.

Type 3: Municipalities whose density is produced by a combination of IF and RW, being the OF low. They are: a) sufficiently attractive in residential terms to have a resident population; b) sufficiently attractive in employment terms to retain part of their working population; and c) sufficiently diverse to attract workers from elsewhere, i.e. to employ people with professional profiles different from those of their own residents establishing bidirectional linkages. Municipalities containing all three traits would be definitively more close to the concept of complex centralities and thus to mature or metropolitan subcentres. Thus by combining the concept of density with basic, but meaningful, elements of mobility it is possible to redefine the way to understand urban density.

4. A proposal for calculating compound density

A way needs to be found to incorporate those meaningful essential elements (RW; IF; OF) of metropolitan mobility (i.e. territorial influence) on a compound density. In Table 1 two families of integration methods are presented⁷; in the first family no information is lost, in that all the elements to be considered are integrated in the compound density; in the second family (semi-integrated), part of the information is lost.

Table 1. Selected alternatives for compound density

Fully-integrated	Multiplicative	$CDm = \frac{\prod_i d_i}{a}$ <p>where d_i may be: RW; IF; OF a=urbanised area</p>
	Geometric mean	$CDg = \frac{\sqrt{\prod_i d_i}}{a}$ <p>where d_i may be: RW; IF; OF a=urbanised area</p>
Semi-integrated	Principal component analysis (PCA)	As habitual, where original variables may be: RW/a; IF/a; OF/a a =urbanised area
	DP2 subfamily	$CDdp2 = \sum d_i / \sigma_i \left(-R^2_{i.i-1,\dots,1} \right)$ <p>where</p> $d_i = x_{in} - x_{im} $ <p>i= RW/a; IF/a; OF/a a=urbanised area see the way to calculate it further in the text</p>

Source: Own elaboration.

⁷ All of the presented methods have been empirically tested using data for Madrid and Barcelona Metropolitan Areas.

In CDm, if only RW and IF are taken, thus it does make sense to positively integrate OF since they represent the incapacity to retain workers which is considered negative and priority is placed on the municipalities with RW=IF. That is to say, those municipalities in which density is equally integrated by resident workers and incoming flows. Nevertheless this approach has two problems: a) it places priority on large municipalities; and b) it makes no distinction between municipalities where density is a result of in-commuting (Type 1 as discussed in Section 3.2), and those where density is a result of the resident workforce being employed in the same municipality (Type 2). By taking the geometrical mean of the numerator (CDg) the first problem, but not the second, is corrected.

On the second family PCA, when working with 3 correlated variables (as is the case), the information can be partially resumed in one variable. Nevertheless, this process has the problem that emphasis cannot intentionally be put on placing priority on any element of metropolitan mobility (i.e. original variable), and it synthesises the information in an imperfect way.

Finally the DP2 subfamily (CDdp2) proposes the construction of a set of combined elements where the impact on the final results depends on the order in which each element is introduced in the calculus process. So, if RW is introduced first, the resulting DP2 will give greater importance to this element. By alternating the order of the introduction of the elements, it is possible to distinguish different types of employment sub-centres.

Originally devised by Pena (1977) distance P2 (DP2) allows for the integration in one synthetic indicator of several partial correlated variables. The starting point is the distance between each geographical area (in terms of one variable, in this case a specific kind of density associated to one element of metropolitan mobility), represented as d_i . That is to say it measures the distance in terms of element i , between municipality n and municipality m which has the lowest value of x_i . So the higher d_i the higher the difference of n municipality in relation to the less dense m municipality in terms of element i . By dividing d_i by its standard deviation the process allows for the combination of original variables expressed in different units, at the same time that differences on the dispersion of data are corrected by expressing the resulting value in terms of standard deviations. $R^2_{i.i-1...1}$ is the determination coefficient from regression of x_i on x_{i-1} , x_{i-2} , ..., x_1 , and as usual expresses the variance of x_i explained by x_{i-1} , x_{i-2} , ..., x_1 . The introduction of this element allows for eliminating the redundant information of a given element contained in the previously introduced variables. In the original way used by Pena the weights of the partial indicators, produced by the order of their introduction, are determined through an iterative algorithm that achieves convergence when the indicator fulfils a set of desirable properties (Montero et al., 2010); nevertheless in our approach the intentionality of the introduction of the partial indicators allows for put apart simple employment concentration from more complex centralities.

In Table 2 the previously discussed properties for each density are depicted for four municipalities corresponding to the three employment sub-centres conceptualised in section 3.2. It should be noted that there are 2 municipalities matching the profile of employment-sub-centre type 3, the only difference between them being the mass of jobs. Supposing for the moment that all four municipalities can be considered as employment sub-centres, classical density is unable to distinguish any difference between the municipalities, despite the fact that

Type 1 is unable to retain 83% of its working population, and Type 2 attracts only 11% of its jobs from other municipalities. Multiplicative compound density (CDm) prioritises Type 3 municipalities, in which there is a certain balance between RW and IF (self-sufficiency is 67%). Nevertheless a) it is biased by employment mass (placing priority on Type 3-larger municipalities over Type 3); and b) makes no distinction between Types 1 and 2 municipalities. Clearly it does not have the same meaning in terms of metropolitan structuring capacity, being almost self-sufficient or almost insufficient; and at the same time, in urban terms, being able, or not as the case may be, to retain working population. By taking the n-root of the numerator (i.e. calculating the geometrical mean) in CDg, the first (scaling) problem is solved but not the second.

Table 2. Employment sub-centre differentiation for classical density and selected alternatives of compound density

Type of employment subcentre	Resident workers (RW)	Incoming flows (IF)	Outgoing flows (OF)	Urbanized area (a)	Self-containment [RW / (RW+OF)]	Oberture [OF/(RW+OF)]	Self-sufficiency [RW/(RW+IF)]
Type 1	100	800	500	100	17%	83%	11%
Type 2	800	100	700	100	53%	47%	89%
Type 3	600	300	100	100	86%	14%	67%
Type 3-larger	900	450	150	150	86%	14%	67%

Type of employment subcentre	Compound densities					
	Classic density [(RW+IF)/a]	Fully-integrated		Semi-integrated		
		CDm [RW*IF/a]	CDg [(RW*IF)^0,5 /a]	PCA [RW/a; IF/a; -OF/a]	CDdp2a [RW/a; IF/a; -OF/a]	CDdp2b [IF/a; RW/a; OF/a]
Type 1	9,00	800	2,83	1,43	1,1	1,7
Type 2	9,00	800	2,83	0,90	1,1	1,6
Type 3	9,00	1.800	4,24	0,27	1,9	0,8
Type 3-larger	9,00	2.700	4,24	0,27	1,9	0,8

Source: Own elaboration.

On the family of semi-integrated methods, PCA places priority on those municipalities in which the element most correlated with PC1 is most present. In this case it is the Type 2 municipality that has the higher density of RW, despite the fact that it attracts virtually no workers from other municipalities (indicating a self-sufficiency of 89%). Finally the CDdp2 sub-family allows for a distinction to be made between the three conceptual employment sub-centres, at the time as being sufficiently robust to scale bias. If RW density is taken first (CDdp2a), the Type 1 municipality is eliminated; if IF density is taken first (CDdp2b) the Type 2 municipality is eliminated. By succeeding on both composed densities (CDdp2a + CDdp2b), it is possible to detect what in this paper is argued to be a metropolitan sub-centre (i.e. Type 3 employment sub-centre).

5. Testing DP2 density in Barcelona and Madrid as an alternative to classical density

5.1 Study areas, data and methodology

The efficiency of the proposed density was tested at the municipal level in the Metropolitan Region of Barcelona (MRB) comprising 161 municipalities⁸. The jobs obtained from the 2001 Census mobility matrix were used as an approximation of the location of economic activity. The data were classified according to 2 digits of the Spanish Standard Industrial Classification of Economic Activities and 1 digit of the National Occupation Classification. Effective (net) urban land was derived from a semi-automatic process of remote sensing using SPOT imagery (see Al Haddad et al., 2006) from the year 2000. The distance between municipalities was measured using a TransCAD analysis of effective road networks.

For the purpose of verifying the conclusions, the analysis was repeated in the structurally different metropolitan region of Madrid⁹ (MRM), which is basically a monocentric urban system compared with the more polycentric structure of Barcelona (MMAMB, 1995; ATM, 1998; Burns et al., 2001; CPSV, 2001; Roca et al., 2009). For both metropolitan areas the central urbanised continuum was identified using the very common criteria of 200 m. Urbanization gap. Table 3 contains the figures (of main data used here) of both metro areas¹⁰.

Table 3. Main characteristics of metropolitan areas of Barcelona and Madrid

Metropolitan area	Number of municipalities	Population 2001	Employment 2001	Urbanized area (sq. km)	2001 Mean population + Employment density
Barcelona	161	4.387.536	1.853.269	548	11.379
Madrid	141	5.448.936	2.284.142	755	10.240

Source: Own elaboration (INE + self-processing SPOT imagery).

The methodology has consisted of:

- a) Calculating the classical density and CDdp2 sub-family as stated above.
- b) Using these densities to identify sub-centres controlling the effect of proximity to the CBD, namely municipalities exceeding 1 std. dev. of regression residuals are considered as potential

⁸ The administrative area for regional planning - "Barcelona Metropolitan Region" - has 164 municipalities, but Castellterçol, Castellcir and Sant Quirze Safaja have their main functional vinculation with other municipalities lying outside the metropolitan area; for this reason they were excluded from the analysis presented here.

⁹ The metropolitan system of Madrid varies depending on the delimitation methodology used: according to NUREC, in 1996 it had 14 municipalities in a surface area of 1,185 km²; according to GEMACA (1996) it had 136 municipalities in a surface area of 6,239 km². The GEMACA delimitation (1996) has been used for this research plus four municipalities from the province of Guadalajara that are clearly metropolitan in physical and functional terms: Alovera, Azuqueca de Henares, Guadalajara and Cabanillas.

¹⁰ See García-López (2007) for an excellent description of the economic structure of the MRB, Muñiz *et al* (2003) and Sánchez (1998) for an historical analysis, and Marmolejo and Stallbohm (2008) for an analysis of urban land patterns. For MRM see de Santiago (Urban, 12; 13).

sub-centres¹¹. The model used is that directly derived from the theoretical discussion in Section 2.1 the linearized formulation of which is as follows:

$$\ln D_i = C + \beta_1 D_{CBD-i} \quad [1]$$

Where:

D is the density at municipality i ,

C is the constant and it is supposed to be the LN of density at distance 0 (i.e. CBD);

and D_{CBD-i} is the distance from i to CBD.

The central municipalities of Barcelona and Madrid are also introduced¹² since beyond containing their respective CBD, the remaining area of the central municipalities may contain sub-centres (although integrated in the central continuum). According to the theoretical discussion it is expected to empirically find a negative coefficient for B_1 , representing the semi-elasticity of density on variation of distance to CBD.

c) Testing the efficiency of the sets of sub-centres identified using the different densities approach by means of different indicators of centrality and metropolitan linkage. The main indicators are as follows:

- The location coefficient (LC)¹³ of top qualified information activities (managers, professionals, scientists and intellectuals) coming from employment following the National Occupation Classification (CNO) as suggested by Marmolejo and Roca (2008). LC of Finance, Real Estate and business support (FIRE and Business) and manufacturing sectors coming from employment following the Spanish Standard Industrial Classification (CNAE in its Spanish initials). Central municipalities, are expected to specialise in both top qualified information activities and FIRE and Business (i.e. services with a rigid demand), and not in manufacturing industries. According to Central Place Theory, centrally distributed goods and services imply transportation costs for consumers and consequently have an impact on market area conformation and land values.

¹¹ McDonald & Prather (1994) consider potential sub-centers as those areas with residuals significant at 95% level of confidence. Nevertheless when analysis units are municipalities using that criterion for the case studies presented here almost 1 of 3 municipalities have residuals above such a limit. Roca *et al* (2009) have found that 1 std. dev. threshold performance quite good for Barcelona.

¹² In Spain for 2001 there is a lack of information about employment location at sub-municipality level, the internal distance of each municipality is calculated supposing that employment is distributed in a concentric internal ring equally accessible from the centre and periphery as follows:

$$A_i = \sqrt{\frac{a * 0,5}{\pi i}} \text{ where } a \text{ is the urbanized area of } i \text{ municipality.}$$

¹³ The location coefficient is calculated as follows:

$$Lc = \frac{\frac{(RW + IF).xi}{(RW + IF)i}}{\frac{\sum_i^n (RW + IF).xi}{\sum_{ix}^n (RW + IF).xi}}$$

Where $(RW+IF)$ is localised employment, x is a given industry of economic activity and i is a given municipality of the metropolitan system.

- Self-sufficiency and self-containment¹⁴ computed using the 2001 Census residence-work data. If a given municipality were to maximise both indicators it would be autarchic in labour market terms (i.e. residents would live and work in the same municipality and the labour force would be supplied locally).

- The integrated interaction value¹⁵, composed by the sum of the interaction value extensively used in the works of Roca y Moix (2005) and Roca et al. (2009) and Coombes and Openshaw (1982). Integrated interaction value measures, using travel-to-work flows, the linkage of a given municipality with all the remaining municipalities in the metropolitan system. The greater this indicator, the greater the reflexive relationship between the given municipality and the rest of the metropolitan system.

d) Testing the capacity of the sets of sub-centres to modify, beyond the effect of CBD, the overall density function of both employment and (working) population, using a polycentric model further discussed in point 5.3.

5.2 Identification of potential sub-centres by means of classical and compound densities

In Table 4 and Figure 1 the main results for Barcelona are reported. As can be seen classical density suggests a large number of sub-centres (23). All the expected sub-centres deriving from local knowledge are included in this set including Sabadell, Mataró, Terrassa, Granollers, Vilafranca and Vilanova, all of which (with the exception of Granollers) are cities which experienced endogenous growth, especially in the 19th Century, and which were integrated relatively recently within the Barcelona functional system by the process discussed in section 2.1. In this list dormitory cities located next to the central municipality of Barcelona are also present, where large housing estates, and industrial parks, were developed in the dictatorial era such as L'Hospitalet and Cornellà, as well as decentralising employment sub-centres such as Martorell oriented to large factory estates. Nevertheless priority is also placed on municipalities as sub-centres, which doubtfully play any role in either the concentration of economic activity or the metropolitan articulation, such as Santa Margarida, Castellví, Pacs del Penedès, Pla del

¹⁴ Self-containment (Sc) is the ratio between resident workers (RW) in a given municipality and resident working population (i.e. resident workers plus outgoing flows (OF) of workers going to work to other municipalities) of the same municipality. It measures the capacity of a given municipality to retain its working population, as follows:

$$Sc = \frac{RW}{(RW + OF)}$$

Self-sufficiency (Ss) is the ratio between resident workers of a given municipality (RW) and locally employed population in the same municipality (i.e. resident workers plus incoming flows (IF) of workers coming to work from other municipalities). It measures the capacity of a municipality to satisfy the local demand of workers with its own working population as follows:

$$Ss = \frac{RW}{RW + IF}$$

¹⁵ The integrated interaction value is calculated as follows:

$$IIV_i = \sum_{j=1}^n \left(\frac{F_{ij}}{(RW + OF)_i (RW + IF)_j} \right) + \left(\frac{F_{ji}}{(RW + OF)_j (RW + IF)_i} \right)$$

where IIV_i is the sum of the interaction value between an i municipality and all j of the system, F are residence-to-work flows, the rest of the variables are the same explained above. As seen, this indicator is reflexive and robust to the bias that may be introduced by the very different size of municipalities in terms of both employment and resident working population.

Penedès, Montseny, Puigdàlber, most of which are in the most rural areas of Barcelona. In those municipalities self-containment is lower in comparison to the rest, and their high self-sufficiency denotes their poor functional relations.

CDdp2a in which RW is introduced first suggests the existence of 19 possible sub-centres. The basic difference between this group and that previously described is that most of the small municipalities with a poor capacity to retain their working population are eliminated. Also Polinyà, a decentralising sub-centre that has captured the manufacturing and service activity lost by Sabadell, and located in an excellent industrial corridor where the regional government has assembled land, emerges in this list. Sant Andreu de la Barca has the same profile. What is highly significant is the elimination of the two dormitory cities¹⁶ of the central continuum - L'Hospitalet and Cornellà.

CDdp2b in which IF is introduced firstly places priority on 19 municipalities as well. In relation to the set of municipalities discussed in the previous paragraph, some municipalities disappear such as Badalona, located in the central continuum. However other municipalities appear, which import workers such as Barberà and other smaller ones.

If the results of CDdp2a & CDdp2b are combined (i.e. imposing a double evaluation) as suggested in section 4, municipalities with a reasonable mix of RW, IF and OF may be selected. Only 15 municipalities satisfy both evaluations. Despite the fact that a quantitative approach is used to evaluate the results, the improvements are expected to be marginal, since the raw data used to construct both densities is the same. Such improvements can be fully appreciated from qualitative local knowledge.

In this context, the elimination of the dormitory cities of L'Hospitalet and Cornellà is quite meaningful, as well as Badalona, which although not a dormitory city, lies within the central continuum. In addition the elimination of municipalities with less than 3.000 jobs is well appreciated. The fact that Barcelona appears as a sub-centre in all models is not futile, being coherent with the scale of analysis. This suggests that beyond the core of the CBD, which can be situated approximately along Passeig de Gràcia and the western side of the Diagonal Avenue, the remainder of the municipality is denser in employment terms to what is predicted by monocentric models. Such density is not surprising taking into consideration the long-standing tradition in the Mediterranean context of residential multifamily buildings in urban centres, not only in the city core, accommodating a wide range of commercial uses at the ground floor level and office related uses as the first floor level¹⁷.

¹⁶ In this paper a dormitory city has been defined, using a local perspective, as that not considered small in employment terms (>2.000 jobs), but unable to retain 65% or more of their resident population.

¹⁷ The reduction of the coefficient of determination for compound densities is coherent with the process used in their calculation, since the objective of such a process is to differentiate the municipalities (of diverging nature) by altering their densities.

Table 4. Potential sub-centres identified in Barcelona by different densities

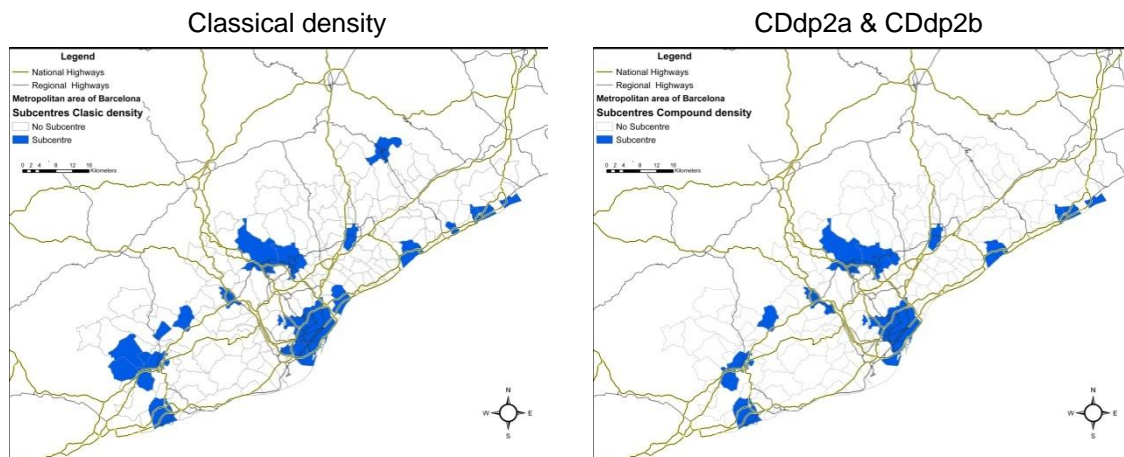
Subcentre candidate	Classic density	CDdp2 subfamily			Employment [RW+IF]	Self-containment [RW / (RW+OF)]	Self-sufficiency [RW / (RW+IF)]
	LN [(RW+IF)/a]	LnCDdp2a (RW; FE; FS)	LnCDdp2b (FE; RW; FS)	LnCDdp2a & LnCDdp2b			
Barcelona	1	1	1	1	779.296	78%	66%
Sabadell	1	1	1	1	69.563	56%	66%
Terrassa	1	1	1	1	67.757	65%	75%
Hospitalet de Llobregat,	1	0	0	0	66.668	31%	49%
Badalona	1	1	0	0	54.998	39%	62%
Mataró	1	1	1	1	42.429	67%	74%
Granollers	1	1	1	1	31.776	51%	41%
Cornellà de Llobregat	1	0	0	0	27.809	28%	35%
Martorell	1	1	1	1	24.749	46%	20%
Vilanova i la Geltrú	1	1	1	1	19.343	59%	73%
Barberà del Vallès	0	0	1	0	17.465	33%	24%
Esplugues de Llobregat	0	0	0	0	15.377	23%	31%
Vilafraanca del Penedès	1	1	1	1	14.031	55%	57%
Sant Joan Despí	0	0	0	0	12.726	23%	25%
Sant Andreu de la Barca	0	1	1	1	12.211	41%	37%
Sant Just Desvern	0	0	0	0	9.702	30%	20%
Polinyà	0	1	1	1	7.298	47%	15%
Pineda de Mar	1	1	1	1	6.806	54%	70%
Malgrat de Mar	1	1	1	1	5.609	59%	67%
Calella	1	1	1	1	5.475	56%	55%
Sant Sadurní d'Anoia	1	1	1	1	5.078	68%	62%
Santa Margarida i els Mon	1	1	1	1	3.561	52%	34%
Canet de Mar	1	1	0	0	2.884	47%	76%
Sant Martí Sarroca	1	0	1	0	768	32%	43%
Torrellaviv	0	0	1	0	584	40%	34%
Castellví de la Marca	1	1	0	0	505	47%	61%
Pacs del Penedès	1	0	1	0	503	23%	16%
Pla del Penedès, el	1	0	0	0	212	35%	65%
Montseny	1	1	0	0	115	60%	70%
Puigdàlber	1	0	0	0	84	28%	54%
Total	23	19	19	15			
Adj R ²	0,26	0,13	0,16	na			
Anova (F)	57,63	23,89	31,43	na			
C	8,15	0,43	0,66	na			
Dist CBD (B ₁)	- 0,03	- 0,03	- 0,03	na			
Dist CBD (t)	- 7,59	- 4,89	- 5,61	na			
Employment density	7.311	7.251	7.311	7.546			
Self-containment	64%	69%	71%	72%			
Self-sufficiency	62%	63%	62%	63%			
Integrated interaction value	0,57	0,60	0,60	0,61			
Central continuum mun.	4	2	1	1			
Dormitory municipalities	2	-	1	-			
Small municipalities	6	2	3	-			
LC FIRE & Business	1,190	1,207	1,223	1,243			
LC top qual. Info. activities	1,136	1,147	1,150	1,169			
LC Manufacturing	0,774	0,783	0,791	0,761			

Source: Own elaboration.

In summary for Barcelona's area compound densities (CDdp2s) allow for placing priority on 15 municipalities, eliminating 8 of the 23 potential sub-centres proposed by the classical density approach: 6 of which are considered small municipalities, with less than 2.000 jobs which is roughly 1% of the metropolitan employment as suggested by García-López (2007), 3 of which are municipalities of the central continuum and 2 are dormitory cities. Moreover, in the CDdp2 set mean employment density increases from 7.311 jobs/sq. km. to 7.546 jobs/sq. km. and self-containment increases 8 points at the same time that self-sufficiency increases by only 1 point, which is considered positive. The mean of the integrated interaction value increases by 4 basic points, indicating a stronger linkage with other municipalities.

All the indicators of urban centrality perform better for the set of sub-centres prioritised with compound densities - these are more specialised in Financial Real Estate & Business activities, as well as in top qualified information activities, and slightly less specialised in manufacturing, as would be expected.

Figure 1. Potential sub-centres identified in Barcelona by different densities



Source: Own elaboration.

In Table 5 and Figure 2 the results for Madrid are reported. Classical density suggests the existence of 23 potential sub-centres of a very divergent nature. This set includes peripheral and mature municipalities such as Alcalá, Torrejón, Guadalajara and Arganda, that may be considered integrated-decentralised sub-centres, and more centric, but beyond the central continuum, dormitory cities such as Fuenlabrada, Móstoles and San Fernando. It also includes, as with the other densities, a lot of small municipalities with less than 2.000 jobs. Such municipalities are basically located in the outer periphery of the metropolitan area lying at a distance more than 55 km from the centre of Madrid, i.e. in the outer 10 percentile. Here there is a crucial point which differentiates Madrid from Barcelona. In Barcelona only 2 of the 7 small municipalities identified as potential sub-centres through classical density lie in the outer 10 percentile ring; and in Madrid 6 of the 8 small municipalities seen as potential sub-centres lie beyond the equivalent limit.

That explains why in Madrid, without taking into consideration which density is used, it is more difficult to eliminate small-outer municipalities since the fitted model in all cases, i.e. exponential negative, is more sensitive to small outer *accidents* in employment density. Of course it may be solved by using different functional forms in each metropolis, but it may obscure the comparison sought here. Thus by using the same functional expression compound density is challenged in the Madrid case.

When CDdp2a is used, in which priority is placed on RW density, the dormitory municipalities of Fuenlabrada, Móstoles and San Fernando are eliminated in relation to sub-centres proposed by classical density, as occurs in Barcelona. Some other municipalities located far from the CBD such as Collado and Ciempozuelos also disappear since the effect of OF - albeit not very strong - reduces the density necessary to succeed as sub-centres in the respective exponential function.

CDdp2b, in which priority is placed on IF density, adds some municipalities which import workers in relation to the previous set of sub-centres such as Humanes, an industrial location of small and compact factories, and eliminates Alcalá due to the double effect exerted by its high self-sufficiency and the distance to the CBD (30 km). Also some non-attracting small and outer municipalities are eliminated, which is considered positive.

There are only 15 municipalities that conjointly succeed after being evaluated by CDdp2a and CDdp2b. What needs to be appreciated is the elimination of the dormitory cities referred to above. Nevertheless compound densities fail to eliminate most of the small and outer municipalities for the reasons discussed above, and unfortunately eliminate Alcalá which, by the use of other methodologies, has showed a local capacity to articulate its surroundings in functional terms.

Table 5. Potential sub-centres identified in Madrid by different densities

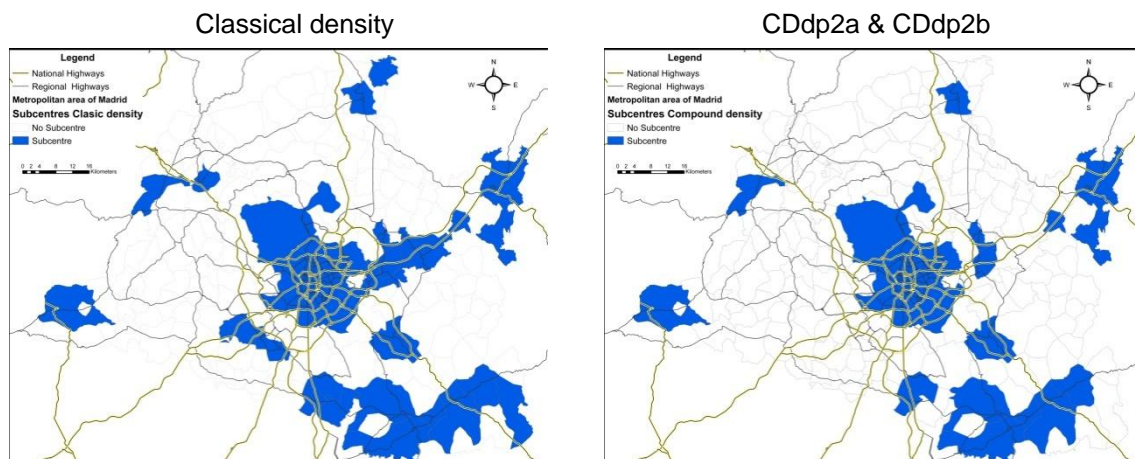
Subcentre candidate	Classic density	CDdp2 subfamily			Employ-ment [RW+IF]	Self- containment [RW / (RW+OF)]	Self- sufficiency [RW/ (RW+IF)]
	LN [(RW+IF)/a]	LnCDdp2a (RW; FE; FS)	LnCDdp2b (FE; RW; FS)	LnCDdp2a & LnCDdp2b			
Madrid	1	1	1	1	1.463.083	87%	72%
Alcalá	1	1	0	0	54.727	54%	71%
Fuenlabrada	1	0	0	0	47.542	29%	47%
Móstoles	1	0	0	0	40.216	30%	60%
Torrejón	1	1	1	1	36.798	44%	52%
Guadalajara	1	1	1	1	27.462	86%	67%
Arganda	1	1	1	1	21.121	69%	49%
Valdemoro	1	1	1	1	16.194	48%	43%
S.Fernando	1	0	0	0	15.575	22%	23%
Collado	1	0	0	0	13.285	40%	61%
Humanes	0	0	1	0	8.856	39%	17%
Azuqueca(Hen)	1	1	1	1	8.820	76%	47%
S.Lorenzo	1	1	1	1	4.895	53%	59%
Ciempozuelos	1	0	0	0	4.169	46%	63%
Ajalvir	1	1	1	1	3.416	32%	10%
Villarejo	1	1	1	1	2.029	62%	67%
S.Martín(V.I)	1	1	1	1	1.830	71%	80%
Chinchón	1	1	1	1	1.501	65%	77%
Colmenar(Oreja)	1	1	1	1	1.313	46%	74%
Torrelaguna	1	1	1	1	1.053	64%	67%
Belmonte	1	1	1	1	257	45%	77%
Estremera	1	1	0	0	170	44%	83%
Valdaracete	1	1	1	1	165	52%	75%
Patones	1	0	0	0	86	51%	69%
Total	23	17	16	15			
Adj R ²	0,37	0,15	0,23	na			
Anova (F)	83,25	25,38	40,49	na			
C	8,14	0,70	1,13	na			
Dist CBD (B _i)	- 0,04	- 0,04	- 0,05	na			
Dist CBD (t)	- 8,77	- 5,04	- 6,36	na			
Employment density	5.351	5.779	5.987	6.053			
Self-containment	76%	83%	84%	84%			
Self-sufficiency	69%	71%	71%	71%			
Integrated interaction value	0,57	0,59	0,59	0,59			
Central continuum mun.	4,0	2,0	2,0	2,0			
Dormitory municipalities	4,0	1,0	1,0	1,0			
Small municipalities	8,0	7,0	6,0	6,0			
LC FIRE & Business	1,069	1,105	1,121	1,125			
LC top qual. Info. activities	1,050	1,072	1,078	1,082			
LC Manufacturing	0,894	0,831	0,813	0,796			

Source: Own elaboration.

Despite the *qualitative* failings described above, the set of potential sub-centres identified by CDdP2 seems to perform better than the set obtained by classical density for a number of reasons.

They are denser in employment terms, more self-contained, have a higher linkage to overall municipalities (integrated interaction value), 2 municipalities located in the central continuum are discarded (Fuenlabrada and San Fernando), 3 dormitory municipalities are erased, 2 small and outer municipalities are reduced, and as a group all the indicators of centrality (LC Fire & Business, top-qualified information activities) are improved, at the same time that manufacturing is reduced.

Figure 2. **Potential sub-centres identified in Madrid by different densities**



Source: Own elaboration.

5.3 Evaluation of sub-centres identified

So far compound densities for both Barcelona and Madrid make small, but meaningful (for locals), improvements in the selection of municipalities as potential sub-centres. Nevertheless it is necessary to check the capacity to modify the overall density function, once the distance to the CBD has been controlled. This evaluation follows the second condition for a sub-centre suggested by McMillen (2001b): “a sub-centre is a site (1) with significantly larger employment density than nearby locations that has (2) a significant effect on the overall employment density function” (McMillen, 2001a: 448-449). Although in theoretical terms such an influence must be expectable, since densities are linked to build structures, their elasticity is somewhat rigid to changes in expanding polycentric systems, as discussed earlier in point 3.1.

In most of the works reviewed the explained density is related to employment, nevertheless the standard urban model explains the distribution of (working) population in relation to employment concentrations. For that reason in this paper we test the efficiency of sub-centres in the explanation of both employment and (working) population¹⁸.

Three different ways have been devised to integrate the effect of sub-centres:

¹⁸ The distribution of the working population (RW+OF) according to residential mobility data (taken from residential variation statistics according to the Spanish population census) is more dependent on proximity to the place of employment, exactly as standard urban model states.

1) The introduction of the distance to the nearest sub-centre as a new term in the monocentric model (expressed in equation [1]). This approach does not have problems of multicollinearity, since each sub-centre is affiliated to only one municipality, but has two inconveniences: a) it is not possible to distinguish the relative importance of each sub-centre; and b) it does not allow to test for overlapping influences coming from different sub-centres.

2) The creation of many variables as sub-centres, each one containing the distance from each municipality to each sub-centre, and introducing them by a stepwise process in the monocentric model. This approach has the advantage that it allows the possible sub-centres to overlap. At the same time, the relative importance of each sub-centre is accounted for by the respective regression coefficient and significance of proximity to each other sub-centre. Nevertheless it is not robust to multicollinearity problem because of the existence of adjacent sub-centres, e.g. Sabadell-Terrassa in the case of Barcelona, and assumes that a sub-centre exerts influence over all municipalities regardless of distance, which in a polycentric framework is implausible.

3) The combination of previous approaches: that is to say, the creation of as many variables as sub-centres, each containing the distance from each municipality to its nearest sub-centre, and setting the distance to the remaining sub-centres as zero, and subsequently introducing them in the general model through a stepwise process. This approach is robust to multicollinearity problem since municipalities are associated to the nearest municipality. At the time that assumes that the effect exerted by the sub-centre is principally local and the relative importance of each potential candidate to the sub-centre is tested in the aforementioned way. This latter approach is that discussed in this paper and its functional expression is as follows:

$$\ln D_i = C + \beta_1 D_{CBD-i} + \sum_j \frac{\beta_{2j}}{D_{SUBj-i}} \quad [2]$$

Where:

D_{SUBj-i} is the distance from i municipality to its respective j -nearest sub-centre¹⁹,

D_i is the density of employment, as used in the majority of the papers reviewed, and working population, as explained previously.

Table 6 contains the results of applying equation [2] in the metropolitan area of Barcelona. As can be seen, in the case of employment density the set of sub-centres deriving from the classical density approach fails to identify any significant sub-centre (at 95% of confidence). On the contrary when sub-centres deriving from the CDdp2 approach are tested, 20% of them (Barcelona, Sabadell and Vilafranca) result being significant and with the expected positive sign, and conjointly improve the variance explained in the metropolitan distribution of employment density. One reason for finding so few significant sub-centres is related to the number of candidates, since the higher it is, the larger the number of fragments in which all the municipalities are organised, and it is possible that an increment in the distance to an ascribed

¹⁹ Note that by introducing the inverse of distance to nearest sub-centre, multicollinearity problems (due to the proximity of some sub-centres to CBD (e.g. L'Hospitalet and Barcelona)) are reduced. However, this expression supports the hypothesis that the effect exerted by sub-centres over the overall density function is lower than the effect of the CBD, so the effect of sub-centres is assumed to be more localised in comparison with a more generalised effect of the CBD, as suggested by McMillen (2003), McMillen (2004) and García-López (2008). An alternative means to resolve this issue is by using factorial analysis to reduce the metropolitan geometry to a non-correlated n-space.

municipality means an approximation to another more important sub-centre. In terms of working population, again CDdp2 sub-centres result more significant than sub-centres deriving from classical density, and marginally improve the efficiency of the polycentric model. Furthermore, CDdp2 sub-centres identified as significant are the very ones which would be selected through qualitative prior knowledge, in terms of the following order of statistical significance: Barcelona²⁰, Sabadell, Vilanova, Vilafranca and Mataró.

Table 6. Influence of sub-centres on employment and working population densities in Barcelona

	Employment density (RW+IF)/a		Working population density (RW+OF)/a	
	Classic density subcentre set	DP2 subcentre set	Classic density subcentre set	DP2 subcentre set
Adj R ² monocentric*	26%	26%	27%	27%
Adj R ² polycentric**	26%	33%	29%	39%
Number of total candidates	23	15	23	15
Percentage of significant subcentres	0%	20%	4%	33%

* Considering only distance to CBD

** Considering distance to CBD and to significant subcentres

Source: Own elaboration.

Table 7 contains the results for Madrid. In Madrid, again the CDdp2 sub-centres are found to be more significant in the explanation of both employment (E) and resident working population (WP). Nevertheless they fail to improve the explained variance of global densities; as a matter of fact in the case of WP the variance is marginally reduced in relation to what is explained by significant sub-centres coming from candidates prioritised by classical density. To understand this it is necessary to analyse in greater detail which sub-centres are found significant. Significant sub-centres deriving from CDdp2 are Guadalajara (E & WP), San Martín (E & WP), Valderacete (E & RW), Torrejón (E & WP), Belmonte (E & WP), Colmenar (E & WP), Villarejo and Chinchón (both only in E). Significant sub-centres deriving from classical density are basically the same, however Torrejón is not considered (in neither E nor WP), and Móstoles, the dormitory city, is added (albeit only in WP). The inclusion of Móstoles, the second most populated municipality in the metropolitan area, by the classical density means the improvement of overall working population density because it self-explains its own working population

²⁰ The fact that Barcelona is included is not banal and it is significant that the effect of this municipality is double, one coming from the CBC contained in it, and the other coming from the rest of the employment located across the city.

density. Nevertheless from a qualitative perspective it is an error to consider such a municipality, unable to retain at least 70% of its working population as a sub-centre. The fact that Mostoles is selected as significant only in the polycentric model when explaining WP and not when explaining E supports this idea. On the other hand, as can be seen with few exceptions (Guadalajara, Móstoles, Torrejón & Villarejo), all the significant sub-centres are relatively small, with less than 2.000 jobs and are located in the outer periphery, so they are some distance from receiving any significant influence from the CBD. Their locally high densities in terms of employment and resident worker population are explained by themselves and for that reason they provide a poor account of the improvement of global density. Nevertheless, the failure in the improvement of overall density is, from a theoretical perspective, not surprising since the elasticity of built-up density is quite rigid in time. For that reason many empirical works have found that the effect of sub-centres is insignificant (Griffith, 1981) or erratic (McMillen and Lester, 2003).

Table 7. Influence of sub-centres on employment and working population in Madrid

	Employment density (RW+IF)/a		Working population (RW+OF)/a	
	Classic density subcentre set	DP2 subcentre set	Classic density subcentre set	DP2 subcentre set
Adj R ² *	35%	35%	34%	34%
Adj R ² **	50%	50%	47%	44%
Number of total candidates	23	15	23	15
Percentage of significant subcentres	35%	53%	35%	53%

* Considering only distance to CBD

** Considering distance to CBD and to significant subcentres

Source: Own elaboration.

6. Conclusions

Polycentrism has become the paradigm of many contemporary metropolises. Some scholars and politicians have seen in this way of spatial arrangement a promising alternative to urban development. Nevertheless, the starting point for the study of polycentrism is the identification of what can be considered as a sub-centre. It implies both the definition of what a sub-centre is expected to be and the devising of indicators capable of their successful identification.

Dealing firstly with a definition, in this paper the conditions stated by McMillen (2001b) are that “a reasonable working definition of a sub-centre is a site (1) with significantly larger employment density than nearby locations that has (2) a significant effect on the overall employment density

function” (McMillen, 2001a: 448-449) are extended by adding two further conditions - (3) the necessity that they represent structural elements within metropolitan systems, and at the same time (4) are attractive places for residential and employment purposes. Such a definition has been argued to be valid when spatial units of analysis are bigger than census tracts or their groupings and when polycentrism basically derives from the integration of formerly independent cities, where the mixture of economic activity and population has historically conformed a paradigm as in Europe (especially in Mediterranean urban centres). By having resident population central municipalities build a local identity, develop cultural services and establish bidirectional relations that are added, as externalities, to the influence exerted by employment concentration, and thus reinforce the structuring of surrounding territories. If metropolitan sub-centres are expected to be modest alternatives to central municipalities, and not only employment concentrations, it seems reasonable to demand the compliance of the aforementioned conditions.

Nevertheless, the employment density used in literature does not make differentiation between different kind of employment concentrations: the aggregated treatment of the numerator (total number of employees) for calculating normal density does not differentiate the density produced by incoming commuters from other municipalities, from the density endogenously generated by employed residents who decide to work in the same area. This explains why the classical treatment of employment density is unable to identify areas that are triply attractive because: 1) they have residents, 2) they are sufficiently attractive in labour terms to retain part of the employed population, and 3) they are sufficiently attractive (e.g. economically diversified) to attract workers from other areas.

This paper proposes an alternative means of calculating employment density by introducing basic, but meaningful, elements of metropolitan mobility. By doing that, mobility and density, two faces of the same coin, are placed together. This point differentiates what has appeared in published research to date, where methods for identifying sub-centres have relied on the analysis of either density or mobility.

Distance P2 (DP2), developed by Pena (1974), is used to compound the densities formed by basic elements of metropolitan mobility. Therefore, it is possible to partially integrate the densities in one sole indicator: deriving from resident workers (RW), the incoming flow of workers (IF) and that lost by outgoing flows of workers (OF). DP2 allows for intentionally intensifying the role played by each element. If RW density is first introduced in the calculus process (CDdp2a) then dormitory municipalities unable to retain their working population are eliminated; if IF (CDdp2b) is first introduced then isolated municipalities unable to establish functional relations with other municipalities are eliminated. By combining the results of CDdp2a and CDdp2b, selected municipalities are a) sufficiently attractive in residential terms to have a resident population; b) sufficiently attractive in employment terms to retain part of their working population; and c) sufficiently diverse to attract workers from elsewhere, i.e. to employ people with professional profiles different from those of their own residents. *Municipalities containing all three traits would more complex centralities than simple employment concentrations, and thus more near of the metropolitan subcentre concept.*

Using the classical density function, i.e. the exponential negative model, the results of applying compound density (CD) have been compared to those produced by the use of classical density

in Barcelona and Madrid, two areas of a similar size but differentiated by the level of polycentrism. Although modest in quantitative terms, compound density makes meaningful improvements in the prioritising of municipalities as metropolitan sub-centres. In Barcelona, from a qualitative perspective all dormitory-municipalities proposed by classical density as sub-centres are discarded by CD density, as well as small municipalities which doubtfully play any role in the metropolitan scene and others located in the central continuum. From a quantitative perspective CD sub-centres are more significant, and improve the explanation of overall employment and working population densities in the polycentric model, in relation to potential sub-centres deriving from classical density. Furthermore, CD sub-centres are denser, more self-contained, more specialised in services with a rigid demand, i.e. central such as FIRE & Business and in top-qualified information activities, and more functionally linked with the rest of the metropolitan area.

For the sole purpose of ratifying the aforementioned conclusions, the analysis was repeated in the structurally different metropolitan area of Madrid, which has a more monocentric structure than Barcelona. The results seem to reinforce the general conclusions reached for Barcelona, nevertheless the CD fails to eliminate all the small municipalities, and to improve the explained variance of overall density (although CD sub-centres are more significant) in the polycentric model. Reasons for such a failure beneath in the more monocentric structure and the existence of small density accidents in the outer periphery, and consequently in the functional model used. Nevertheless, the failure in the improvement of overall density is, from theoretical perspective, not surprisingly since elasticity of (built) density is quite rigid in the time.

The disaggregation of the essential components of employment density appears to reveal significant information regarding the detection of sub-centres that play a major role in the concentration of employment and the paradigmatic features of central cities in Europe. Nonetheless, the method proposed here is not exempt of criticism, hopefully too it will help to stimulate the debate about the most appropriate approach to conceptualise/identify mature subcentres in polycentric Mediterranean metropolises.

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