# ACE 40

# VAN DER LAAN AND MESSIAEN'S CREATIVE FREEDOM FROM A SYSTEM

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## Structured abstract

### Objectives

An increasing number of artistic expressions around the world give the impression that creation has a systematic root based on the definition of elements and their interrelation. The aim of this study is to investigate the nature of these systems in both architecture and music and stress that creator's freedom is not diminished by their use.

### Methodology

Two well-known authors –the architect Dom Hans van der Laan and the composer Olivier Messiaen– are analysed through their own work systems and the term "universal necessity" is used to define their work problems solved by mathematical methods.

### Results

The results show that creative freedom can be reached from order, and indicates that knowing the rules of the discipline's system is a way to achieve creative freedom.

### Originality

There is a centuries-long unresolved controversy in discussion of creativity and aesthetics focussing on the question: Does a set of rules, such as the rules of counterpoint, stimulate or inhibit the expression of artists? This paper wants to contribute to this debate by adding two examples which are independent but also related. This research is addressed to scholars, art and architecture historians and academics in both disciplines architecture and music.

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# 1. Introduction

The remarkable variety of artistic manifestations all around the world and in all times generates considerable interest in studying the nature of the creative process. The fact that this huge amount of work is conceived by the human brain suggests the possibility of a systematic nature of human creativity (Cohen, 2000; Monaghan, 1968). A system is a basic structure composed of some elements and the relations among them (Deaño, 1999). However, it is commonly believed that following a system's rules leads to a lack of creative freedom.

This phenomenon affects all artistic production: from the most intangible of the arts (music) to the most material of them (architecture). An analysis on all artistic disciplines would be a titanic task and that is one of the reasons for choosing architecture and music as study cases. Furthermore, the significant amount of literature defending their interrelation attracts widespread interest in the comparison of both disciplines. (Wittkower, 1998; Moreno Soriano, 2008; Bofill, 1975) However, the real reason for putting together architecture and music is the fact that neither need to reproduce the real world, in the same way as literature, theatre or painting. They possess a non-linguistic scaffolding within a coherent system. In other words, architecture and music do not use linguistic signs but rather composed designs where the interrelations constitute ordered systems.

In the recent years, it has been generally accepted that the artist can do whatever they want in order to achieve their desires and this leads to arbitrariness. For example, Rafael Moneo showed that there is an increasing number of architects that found their project decisions to be arbitrary (Moneo, 2005) and, previously Igor Stravinsky illustrated that today's artist is somebody who speaks their own language and nobody understands them (Stravinsky, 1970). More work is needed to explain that artistic disciplines have a systematic nature and to defend that its knowledge brings a major control of the discipline itself.

This paper analyses two artistic creators that used a coherent system in their work: the composer Olivier Messiaen and the architect Dom Hans van der Laan. By analysing their work, we can illustrate the systematic nature of the disciplines that provides the tools for the freedom achieved in their creations. Our research reveals through particular examples from both authors that those systems helped them to reach their creative purpose.

# 2. Between pure mathematics and aesthetics

There seem to be three possible approaches to the relation between art and mathematics. The first one can be named as Pure Mathematics, in which mathematics solves problems that are unlinked to the artistic interests and mathematics covers the entire question. The second one can be named as Universal Necessity, in which the problems of art have an explanation in mathematics and the mathematical solution is artistically interesting. The third one considers the problems of art as a question of pure taste and there is no place for the mathematical or logical reasoning, known as Aesthetics.

Nevertheless, we announce that our paper is not going to base its investigation in the first approach neither the third one. It is the second one, the conjunction of an artistic necessity and a mathematical way of resolution of this necessity, what we find necessary to explain.

## 2.1    Pure mathematics

Mathematics are intended to describe many of the properties and processes in the world; its language, however, must be precise to the extent of appealing to the human reasoning rather than to sensible experiences. In its more rigorous and toughest expression, modern mathematics rely on the Axiomaic System of Zermelo-Fraenkel (with Choice) which is stated in first-order logic language.

In this setting mathematics only cares about "what is true", without ever considering "why" or "for what purpose". The following example will clarify this perspective.

Suppose we are given the following mathematical issue (Figure 1): "Find the ratio of a rectangle that may be decomposed into a square and smaller rectangle with the same (reversed) ratio."

Figure 1. **Both grey rectangles are proportional**
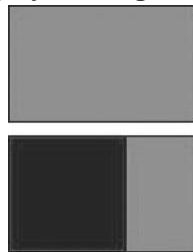


Source: Juan José Madrigal

Even when this issue is not stated in terms of this rigorous language, it may be translated into such terms, and the answer would be: there exists exactly one (bigger than one) real number

$$x = \frac{1 + \sqrt{5}}{2}$$

For the ratio of such rectangle, which is the fundamental golden number $\phi$.

Now suppose we are given another slightly different mathematical issue (Figure 2): "find the ratio of a rectangle that may be decomposed into two squares and smaller rectangle with the same (reversed) ratio."

Figure 2. **Both grey rectangles are proportional**



Source: Juan José Madrigal

*Josep Llorca, Juan José Madrigal*

And again, there is a precise answer: there exists exactly one (bigger than one) real number

$$x = 1 + \sqrt{2}$$

For the ratio of such rectangle.

Both numeric solutions above studied solve the pure mathematical statements that were asked. They are the unique solutions to their problems. Moreover, these numbers hold natural and artistic significations, and they are included in the family of metallic means, as is widely explained in Spinadel (1998; 1999). For that reason, we can clearly see that in this point there is already a necessity that is covered by these numbers. Hence, they do not serve only to the "pure mathematics", but they cover a necessity, that we name as "universal necessity".

## 2.2    Universal Necessity

If the first approach to the relation between art and mathematics is pure mathematics, as we have just seen in 2.1, the second step links art and mathematics by a necessity that nature needs to solve. Therefore, this second approach deals with a natural necessity. This natural necessity only can have one possible answer. When a natural necessity falls into a unique solution we can treat it as an objective necessity, because the result follows the only possible way. Some literature has been written in this field and every author uses a different term to describe this reality. Doyal and Gough prefer the terms "objective" and "universal" to describe human necessities: "objective" because its theoretic and empiric specificity is independent to the individual preferences and "universal" because its concept is the same for all cases (Doyal & Gough, 1991). Kant also speaks about an "objective necessity" as the necessity that remains along the time (Kant *et al.*, 1998). In the case of art and mathematics, we cannot consider artistic necessities as natural necessities, because art is artificial, and this term can lead to confusion. But once the necessities of art are treated by a universal approach, we can call these necessities as "universal necessities".

There are lots of examples where we can find a "universal necessity" translated into a mathematical property. However, not every mathematical property (discussed in 2.1) may come from a "universal necessity".

Despite the fact that we are going to centre our attention into the relation between art and mathematics, we find it interesting to show an example where nature and mathematics link into such a "universal necessity". After this example we will explain the "universal necessity" in art and mathematics.

The golden number

$$\phi = \frac{1+\sqrt{5}}{2} \approx 1.618$$

has many demonstrations in nature: flower petals, seed heads, shells... One may refer to the following excellent video[i].

As shown in the video, plants usually pack their leaves or seeds in arrangements involving the golden ration. This is studied in phyllotaxis and is a most known effect, whose reasoning includes both an evolutionary advantage and mathematical property. Let's discuss this phenomenon further.

Plants can grow new cells in spirals (Figure 3); the spiral happens naturally because each new cell is formed and put after a turn. But the angle of the turn made before putting a new cell is crucial. Suppose a plant puts a new cell with an angle equal to 2/5 a complete turn, that is, (2/5) x360º = 144º.

Figure 3. **Spatial distribution for α=2/5**
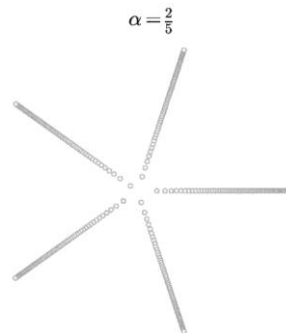


Source: Juan José Madrigal

This is rather a bad distribution for the seeds to be equally spaced and profiting the sunlight. If we take α to be the ratio of a complete turn to be made, then α should not be a rational number, otherwise inconvenient branch patterns will arise.

We need some irrational number, what about α=π (Figure 4)?

Figure 4. **Spatial distribution for α= π**



Source: Juan José Madrigal

That is not good either. One may notice the formation of seven curved branches. Why? Well, we must remember that π is really close to 22/7

$$\pi = 3.1415 \ldots$$

22/7 = 3.1428... So similar behavior is to be expected! π is not equal to 22/7, so the branches are not straight, but curved.

How can we realize that a number is close to very simple ratios? Mathematics has a concise language to this issue: continued fractions:

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{243 + \cfrac{1}{1 + \cdots}}}}}$$

The process is quite simple: we subtract the whole part, and the decimal part (less than 1) is then equal to 1/b, for some real number b greater than one, and so on. Large numbers imply a nice approximation by truncating the continued fraction:

$$\pi = 3 + \cfrac{1}{7 + \cfrac{1}{15 + \cfrac{1}{1 + \cfrac{1}{243 + \cfrac{1}{1 + \cdots}}}}} \approx 3 + \frac{1}{7} = \frac{22}{7}$$

Then there should be small numbers appearing in the continued fraction of α! And the best candidate for such a job is the golden number
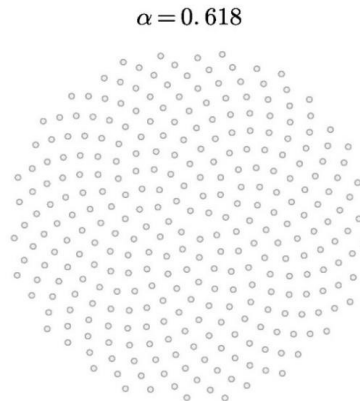
$$\phi = 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}}} \approx 1.618$$

Or its inverse

$$\phi^{-1} = \phi - 1 = 0 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}}}} \approx 0.618$$

And it works (Figure 5)!

Figure 5. **Spatial distribution for α= ϕ, the golden number**

$$\alpha = 0.618$$



Source: Juan José Madrigal

Here the universal necessity was the best distribution for the seeds to be equally spaced and profiting the sunlight.

## 2.3 Aesthetics

Finally, the third approach to the relation between art and mathematics is "aesthetics". A lot of work in the field of aesthetics of mathematical forms has been written (Montano, 2016; van Gerwen, 2011; Zeki *et al.*, 2014; Field, 2012). We are not going to focus in this insight here. Nevertheless, our aim in this article is to highlight the procedures in which art and mathematics help each other solving "universal necessities". As we have announced before, it is the second approach that we are interested in.

## 3. Background

As the two study cases belong to different disciplines: architecture and music, we must do a concise framework on the relation of them. A big amount of work about the relation between architecture and music has been written since the treatises (Vitruvius 2001; Alberti 1988) to the modern architects and composers (Xenakis, 2008; Sheridan & Van Lengen, 2003), passing through philosophers (Trías, 2003) and art historians (Semper, 2004; Wittkower, 1998). Some of the writings argue how difficult the reconciliation of both disciplines is (Scholfield, 2011), whereas others show the similarities as the basis for a unified insight (Wagner, 1993).

Much work hast been done in the field of recognition of the treatises (Pintore, 2004; Costantini, 2013) and in the regarding of modern artists (Gómez-Collado *et al.,* 2017). In particular, there are many investigations around the figure of Van der Laan and his proportional system (Voet, 2012; Dewitte, 2015; Voet, 2016). Similarly, a big amount of investigations about the figure of Messiaen and his combinational system of notes have been arisen since its invention (Freund *et al.*, 2015; Bigo *et al.*, 2011). Nevertheless, the information regarding these two authors under the same consideration -the mathematical view- is inexistent and this paper aims to cover this hole and use this argument to defend that creative freedom can be reached from a systematic approach.

## 4. Methodology

The methodology used in this paper consists of describing the systems used by Van der Laan and Messiaen, and the verification of their creative freedom in decision making. For this purpose, we show the characteristics of both systems in order to be aware of the application in their works. Subsequently, we analyze their application to personal works: Saint Benedictusberg Abbey new church and 8th movement of Turangalîla Symphony. Finally, we show those points where the system has helped them or not to the creative freedom.

On one hand, Van der Laan used his Plastic Number system in the construction of Saint Benedictusberg Abbey new church. Firstly, an enumeration of the strengths and weaknesses of this system is showed in the first part of the article. Secondly, a detection of the points where the system guided him to build the church is shown. Finally, we indicate those points where the system specially helped him to reach the creative freedom.

On the other hand, Messiaen uses his Limited-transpositional Modes system in the conception of the 8th movement of Turangalîla Symphony. Again, an enumeration of the strengths and weaknesses of this system is showed in the first part of the article. Secondly a detection of the points where the system guided him to conceive the movement is outlined. Finally, we indicate those points where the system specially helped him to reach the creative freedom.

The comparison between the two systems in this article allows us to explain that the two disciplines (music and architecture) share a systematic nature. Moreover, with this method, it is easy to sum up that two different disciplines possess different systematic roots and, at the same time, this systematic root can be seen as the link between them. Not only differ one from the other but also can be compared through this systematic view.


## 5. The systems of Van der Laan and Messiaen

### 5.1 The universal necessity of Van der Laan

Dom Hans Van der Laan dropped out his architecture degree in 1926, when he was in the third year in Delft. He decided to consecrate his life to God as a Benedictine monk and, at the same time, aimed to search the "primitive origins of architecture: to that fundamental link between the technical building art and our need of order and definition in the space around us" (Padovan, 1994, back page). His discovery of the Plastic Number must be related to his fascination with measurements and proportions since he was at his father's architecture workshop (Padovan, 1994). The major problem he looked for was the possible division of a segment AB by C and the subsequent division of BC by D in which AB, AD, BC, AC, CD and BD were in a continuous proportion. Furthermore, the Plastic Number is determined by, and contained within, two limits: an inferior limit in which the difference between two measurements cannot be distinguished by human eye, and a superior limit in which two measurements are so different that they cannot be compared.

*Josep Llorca, Juan José Madrigal*

The Plastic Number system is flexible enough because it defines a scale of measures from a given measure but does not allow to establish a fixed grid (Corcuff 2012). In this aspect, Van der Laan's system is similar to Le Corbusier's. But, unlike Le Corbusier's Modulor system, Plastic Number system consists of a proportion and not a fixed scale (Padovan, 1994). For this reason, the Plastic Number must be used carefully because of its most characteristic aspect: the ability for measuring continuous world into a discontinuous system. This fact can, hence, fall into total permission of measuring and into total restriction of measuring.

We have already mentioned that Van der Laan was searching for the possible division of a segment AB by C and the subsequent division of BC by D in which AB, AD, BC, AC, CD and BD were in a continuous proportion. We all know that this statement belongs to the pure mathematics field. But we can also know that this statement is the formalization of a real problem that Van der Laan expressed in one of his letters (Padovan, 1994). Speaking to architect Richard Padovan about how was he searching for this proportion when he was young. He had been fascinated by problem of proportion since childhood. He wondered where the measures of the buildings came from.

Above all, I sought the origin of these measures, but they vanished into the vagueness of the design sketches, which by that time were mostly done by my elder brother… The course at Delf had taught him no fixed norms for design, just as I would find myself. In the BSK we tried to discover them for ourselves, and the following story belongs to that period.

On Sundays my father smoked better cigars than during the week… Inside the lid of the cigar-box was a view of the Domtoren (cathedral tower) of Utrecht… For much of my boyhood this image served as a model for my play with building-blocks. I had never been to Utrecht, but when I heard that my uncle's family had moved there, I immediately asked if I could come and stay. I wanted to know how the tower was composed, so as to use it as point of departure for my own designs. For two days I sat from morning to evening in a corner of the cloister, and I finally hit on the very regular and simple scheme… next day I got into the works office for the restoration of the choir which was then under way, and was able to examine the drawings of the tower, complete with measurements which corresponded wonderfully with the scheme I had mapped out… in with all measures were multiples of each other. I several times spoke about this to the BSK group, but always came in conflict with the Prof, who interpreted the balustrades as intermediate elements uniting the larger parts, whereas I saw them as reflections in the parts of the composition of the whole: and eighth part of each subdivision, just as the roof zone was an eighth part of the whole tower.

(…) And I recognized in each of the three parts its own crowning in the form of the balustrades. This last was something that had fascinated me ever since my childhood: to find in the subdivision of a thing the aspect of the whole.

Padovan notes that the phrases "all measures were multiples of each other" and "reflections in the parts of the composition of the whole" contain the essence of what later became the plastic number, and indeed of Van der Laan's whole theory of design. In fact, this phenomenon should have a formalization in mathematics, Van der Laan thought. This "universal necessity" should have a mathematical expression. Finally, this expression is Plastic Number.

In a lecture, Van der Laan concludes that the plastic number is contained between a geometric and arithmetic conception of quantity.

So we see that the plastic number throws a bridge, as it were, between the two mutually exclusive kinds of quantity, the how-many and the how-much. We can define the ordinance based on this number as an arithmetical order in which attention is paid to certain tolerances. The plastic number bridges the how-many by arithmetic conceptions, and the how-much by geometric conception.

- *Van der Laan and the Plastic Number*

Van der Laan's universal necessity may be translated into mathematical terms as follows: to divide a segment in several pieces, so that the lengths of the resulting pieces (or the union of adjacent ones) fit neatly into a (decreasing) geometric progression.

For this requirement to be studied we will focus first on the simplest examples.

- *One cut and two pieces (Figure 6)*

### Figure 6. **Golden number**
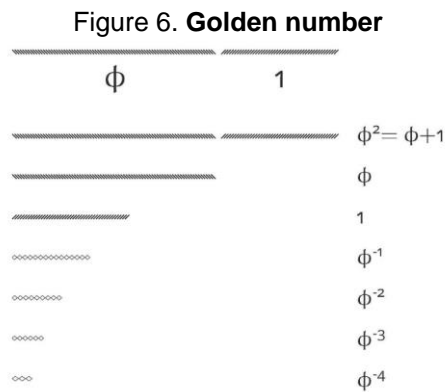


Source: Josep Llorca-Bofí and Juan José Madrigal

Taking as unity measure the length of the shorter piece and $\phi$ the length of the bigger, our defining equation becomes

$$\phi^2 = \phi + 1$$

Whose positive solution is the ubiquitous golden number.

$$\phi = \frac{1 + \sqrt{5}}{2} \approx 1.618$$

When the geometric progression is continued, the new lengths may be arithmetically obtained from larger ones. This is because our defining equation may be multiplied or divided by a power of $\phi$
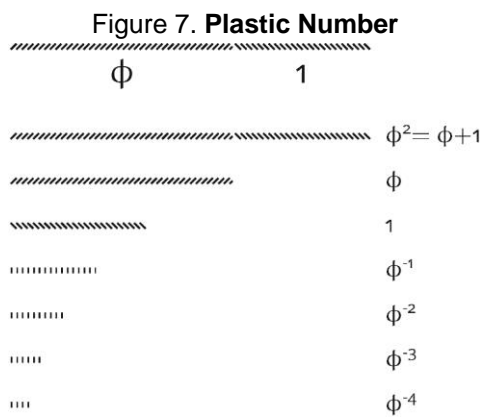
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$$\phi^2 = \phi + 1 \Longrightarrow \phi^{k+2} = \phi^{k+1} + \phi^k, \qquad k \in \mathbb{Z}$$

And thus each new length is the difference of the two previous ones. One may notice that this number satisfies the pair of equations

$$\phi + 1 = \phi^2$$

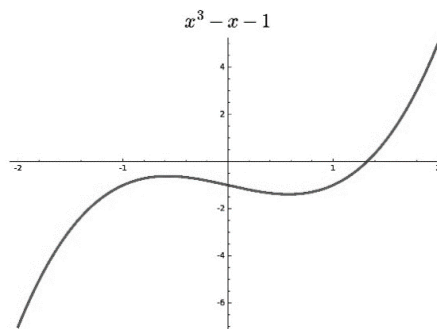$$\phi - 1 = \phi^{-1}$$

- *Two cuts and three pieces (Figure 7)*

Figure 7. **Plastic Number**



Source: Josep Llorca-Bofí and Juan José Madrigal

Proceeding as before, our defining equation becomes

$$\psi^3 = \psi + 1$$

The function f(x)=x³-x-1 has only one real root (Figure 8)

Figure 8. **Root of the function**



Source: Josep Llorca-Bofí and Juan José Madrigal

Whose value is the *plastic number*

$$\psi = \frac{\sqrt[3]{12(9+\sqrt{69})} + \sqrt[3]{12(9+\sqrt{69})}}{6} \approx 1.325$$

As above, when the geometric progression is continued, the new lengths may be arithmetically obtained from larger ones.

$$\psi^3 = \psi + 1 \Longrightarrow \phi^{k+3} = \phi^{k+1} + \phi^k , \qquad k \in \mathbb{Z}$$

And thus each new length is the difference between the third before and the first before. But surprisingly enough, it is also the difference between the fifth before and the fourth before, thus enriching the systems of measure.
Due to these considerations, the following pair of equations is satisfied:

$$\psi + 1 = \psi^3$$

$$\psi - 1 = \psi^{-4}$$

- *More cuts and pieces*

For a division of the initial segment into pieces satisfying our requirement, we first need a ratio η>1 such that

$$\eta + 1 = \eta^m$$

$$\eta - 1 = \eta^{-n}$$

For some integer numbers m and n, because we want that for each pair of consecutive lengths their sum and difference belongs again to our geometric progression. This system, though, has been already studied, and their solutions called morphic numbers. And as matter of fact, it has been proven that there are no morphic numbers greater than 1 other than the golden number and the plastic number (Aarts *et al.,* 2001). This being so, we may assert that the golden and plastic systems of measures are the only ones that both combine geometric and arithmetic relations in the way described above.

## 5.2    The universal necessity of Messiaen

Olivier Messiaen finished his music studies in 1930 at Paris Conservatory and he decided to devote his life to musical composition, musical teaching at Paris Conservatory, and musical interpretation as an organist in Sainte Trinité church in Paris. His discovery of Limited-transpositional Modes must be related to his fascination with the "charm of impossibilities" since he searched continuously for music with plenty of light and color (Messiaen, 1944). The major problem he looked for was the possible arrangement of notes –or what is the same, a scale
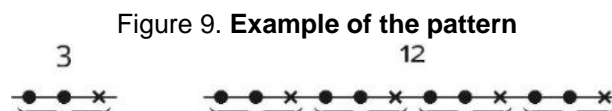
*Josep Llorca, Juan José Madrigal*

which is able to possess not just one centre of attraction but more than one. This search lead him to those scales containing symmetric patterns of notes. In addition, the last note of each pattern is the first of the next pattern. For this reason transpositions of the scales (or modes, in Messiaen's terminology) are limited and, consequently, it encourages the apparition of several centers of attraction inside the mode (Messiaen, 1944). In words of Messiaen (1944): One point will attract our attention at the outset: the charm of impossibilities. It is a glistening music we seek, giving to the aural sense voluptuously refined pleasures. At the same time, this music should be able to express some noble sentiments (and specially the most noble of all, the religious sentiments exalted by the theology and the truths of our Catholic faith). This charm, at once voluptuous and contemplative, resides particularly in certain mathematical impossibilities of the modal and rhythmic domains. Modes which cannot be transposed beyond a certain number of transpositions, because one always falls again into the same notes; rhythms which cannot be used in retrograde, because in such a case one finds the same order of values again –these are two striking impossibilities [universal necessity].

Messiaen recalls the same necessity some pages below (Messiaen 1944): Let us think now of the hearer of our modal and rhythmic music; he will not have time at the concert to inspect the nontranspositions and the nonretrogradations, and, at that moment, these questions will not interest him further; to be charmed will be his only desire [universal necessity]. And that is precisely what will happen; in spite of himself he will submit to the strange charm of impossibilities: a certain effect of tonal ubiquity in the nontransposition, a certain unity of movement (where beginning and end are confused because identical) in the nonretrogradation, all things which will lead him progressively to that sort of theological rainbow which the musical language, of which we seek edification and theory attempts to be.

To sum up, we can recall that the universal necessity of Messiaen consists of some "modes which cannot be transposed beyond a certain number of transpositions" and "rhythms which cannot be used in retrograde". Finally, Messiaen's "universal necessity" considers that the desire of the hearer is just "to be charmed". This universal necessity should have a mathematical expression. And in the case of the modes, which is what we are going to focus on, Messiaen's Limited-transpositional Modes is the solution.

- *Messiaen and the Limited-transpositional Modes*

Messiaen's universal necessity may be translated into mathematical terms as follows: to find all the possible bullet-cross patterns in a row of 12 elements, up to cyclical reordering, that fit into the repetition of a pattern in a subset of 1, 2, 3, 4 or 6 elements. Here (at Figure 9) is presented an example of such a pattern.

Figure 9. **Example of the pattern**



Source: Josep Llorca-Bofí and Juan José Madrigal

Let's start by direct inspection in subsets of size 1, 2, 3 or 4. As stated we consider cyclical reordering as repetition to get the number of genuine patterns (Figure 10).
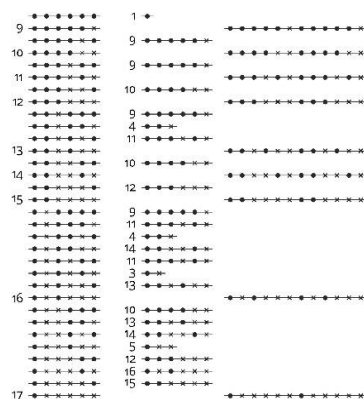
Figure 10. **Permutations for 1, 2, 3 and 4 subsets**
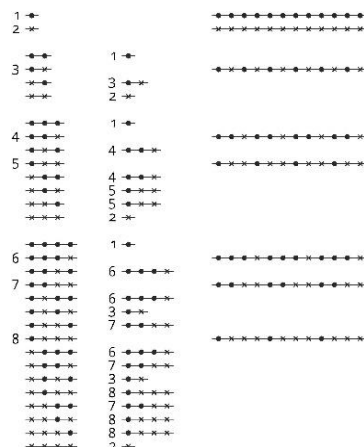


Source: Josep Llorca-Bofí and Juan José Madrigal

For a subset of order 6, we may suppose that our pattern starts with a bullet, since cyclical reordering may place a bullet in the first position (if there are no bullets at all, this is the empty pattern already considered) (Figure 11).

Figure 11. **Permutations for the 6 subset**



Source: Josep Llorca-Bofí and Juan José Madrigal

Now we note that there are less Messiaen's limited-transpositional modes than possible permutations on the mathematical reasoning. In particular, there are 17 possible permutations whereas there are only 7 Messian's limited-transpositional modes. How can it be possible? This is a perfect example of combination of mathematics and art, in which neither pure mathematics nor pure aesthetics form the law, but a balance between reason and emotion.

In fact, only the permutations 3, 4, 6, 10, 12, 11 and 9 correspond to Messiaen's limited-transpositional modes 1, 2, 3, 4, 5, 6 and 7. The rest is considered just as the dodecaphonic scale (1), the null scale (2), the seventh diminished chord (5 and 13) and scales without the necessary number of notes capable of a continuity (7, with too big intervals; 8, with only three notes; 14, 15, 16 and 17, with too few notes)

## 6. How did Van der Laan and Messiaen use their systems

### 6.1    Van der Laan's new church for Saint Benedictusberg Abbey

Van der Laan used the plastic number system of proportions firstly in Saint Benedictusberg Abbey new church, near the village of Vaals. Having been commissioned in 1956, the building was not finished until thirty years after this date. This fact allows van der Laan the chance for the application and verification of this theory day after day and in every stage of the building development (Ferlenga & Verde, 2000). Hence, the entire design of the building is governed by plastic number and the methodology he used when applying the system of proportions can be found in the fourth lesson of van der Laan's *Architectonic Space* book (Laan, 1983): Every form is determined by three dimensions, measured perpendicularly to each other; the smallest of the three early servers as a yardstick. For the sphere all three dimensions are equal; for the cylinder one dimension is enlarged and the other two have the size of the yardstick; for the disc two dimensions are enlarged and only one corresponds to the yardstick. (…) In architecture these forms are squared, just as the ball of clay is given the rectangular form of the brick. We then speak no longer of sphere, cylinder and disk, but of block, bar and slab (Laan, 1983).

Block, bar and slab are the three extremes of a number of forms that Van der Laan arranges in his Morphotek, which is used rigorously in the conception of every space in the church: the central hall, the galleries around it, the courtyard in front of it, the galleries of the courtyard and, finally, the entire building, as we can see in figure 12.

Figure 12. **Each part of the church (right) corresponds to a form in the Morphotek (left)**



Source: Josep Llorca-Bofí

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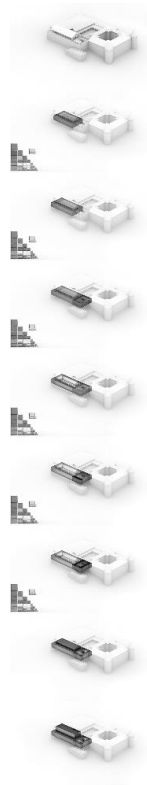
Van der Laan not only considers his system as useful for the big scale of his building, but also he uses it in the measures of the walls, columns and lintels. Every single part of the building is formed under the proportions derived from the Plastic Number system (Figure 13):

Figure 13. **The section along the path beginning from the entrance and finishing into the church, through the crypt and the courtyard**
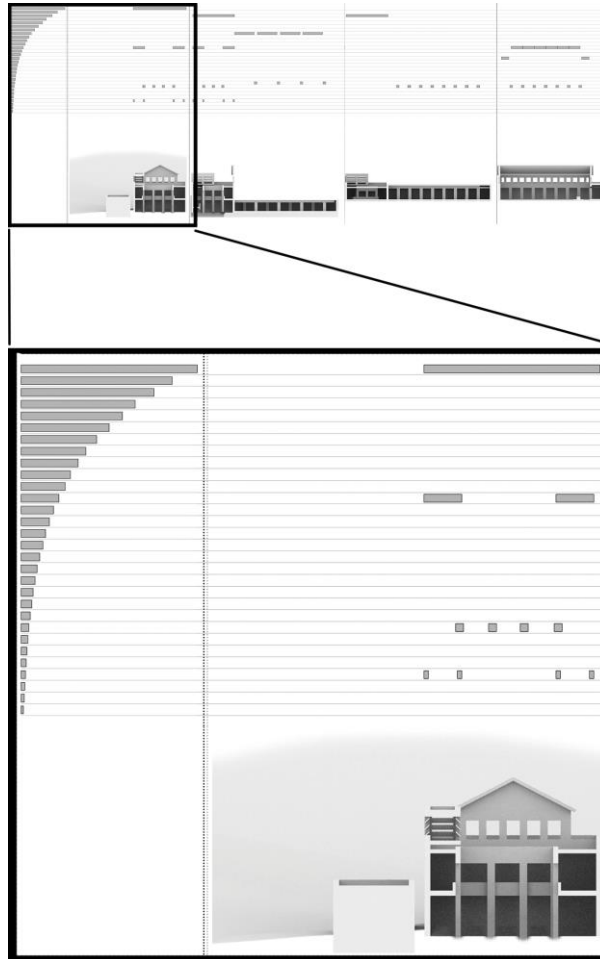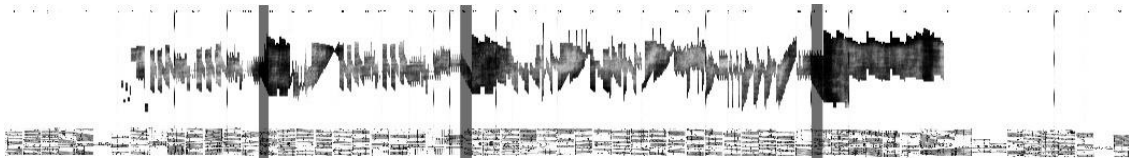


Source: Josep Llorca-Bofí
Note: On the top of the section, every measure of the actual building corresponding to the Plastic Number series (on the left) can be checked. A zoom of the entrance to the courtyard is depicted below.

## 6.2    Messiaen's 8$^{th}$ movement from Turangalîla Symphony

Olivier Messiaen, on the other hand, used his Modes of Limited Transposition in some works before the Turangalîla Symphony. This fact allowed him to have plenty control of the system in this work. Being commissioned by serge Koussevitzky for the Boston Symphonic Orchestra, it was premiered in 1949 under Leonard Bernstein's conduction. Messiaen uses his Modes of Limited Transposition in the whole symphony. For our study, the 8$^{th}$ movement is going to be useful on account of the clarity of its use in the work.

*Josep Llorca, Juan José Madrigal*

As said before, these modes, which are based on the twelve-tone chromatic system, take certain notes from it. These notes form some symmetric patterns in which the last note of each pattern is always the first of the next pattern. This fact leads into the limitation of the number of the mode transpositions and the sense of ubiquity searched by Messiaen in his "charm of impossibilities". There are only seven Modes of Limited Transposition that fulfill these conditions and Messiaen uses them in all his work. In the 8th movement of the symphony, Messiaen uses the 2nd, 3rd and 4th modes in combination of some moments of tonal harmony centered in C, D and F sharp. We can easily hear the presence of the 2nd mode preceding the three entrances of the main theme of the movement (Messiaen 2002). This presence is highlighted by the scales played by the lowest and highest parts of the entire orchestra: the double basses and the flutes, as we can see in figures 14 and 15:

Figure 14. **8th movement of Turangalîla Symphony with three extracts highlighted**



Source: Josep Llorca-Bofí

Figure 15. **First, second and third extracts showing the modes in basses and flutes**
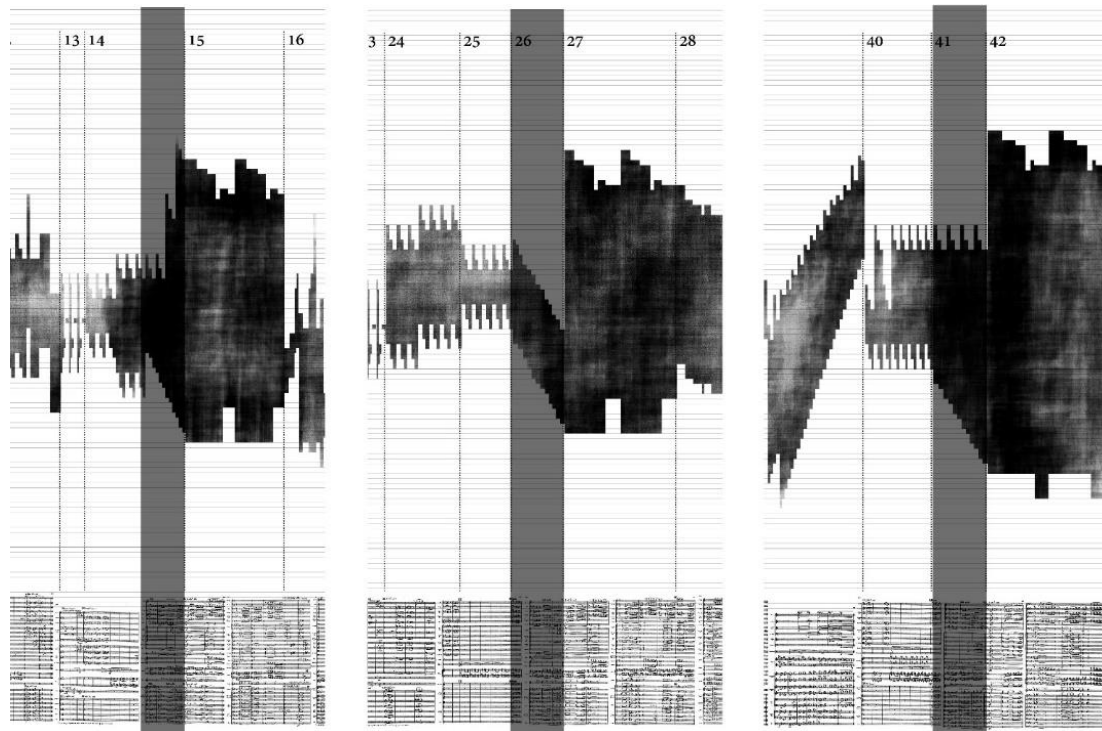


Source: Josep Llorca-Bofí

## Conclusions

As we said before, the common belief is that following a system decreases creativity. But in the case of Van der Laan and Messiaen we can disagree this statement.

Van der Laan uses his plastic number in order to build a bridge between two exclusive kinds of quantity, the how-many and the how-much attending to the universal necessity of discrete measurements of a continuous reality, that is the discrete human measurements into the continuous nature. This necessity, mathematically speaking is translated into a proportion able to combine geometric and arithmetic relations in the way described above. This connection between the continuous world of natural objects to the discontinuous fact of mathematical measurements demands the viewer's attention and is fulfilled by the architectonic building. In other words: his system tends to search a marriage between nature and human being in a way that can include all kind of spatial possibilities, that is, the definition of architecture. This achievement may be seen as a product of the systematic use of the architectonic discipline rather than arbitrary use of some architectonic elements.

Messiaen, on the other hand, uses the system of limited-transpositional modes in order to achieve a bigger range of musical possibilities in his work. The three extracts showed before are followed by three consecutive explosions of tonal harmony centered in C, D and F sharp. This connection between his modal system to the tonal system demands the listeners' attention and fulfills the "charm of impossibilities" created by the system itself. The fact that this system is able to connect with other musical systems brings him more freedom in creative decision-making. In other words, his music tends to cover a wide variety of music harmonies that goes from the serialism to the tonal harmony and the modal scales, all by means of coherence and proficiency. This achievement may be seen as a product of the systematic use of the musical discipline rather than an arbitrary use of some musical elements.

**Author's contributions:** Josep Llorca-Bofí has developed the core idea, the architectural explanation of the plastic number theory, its application to Van der Laan's work, the musical explanation of the modes and its application to Messiaen's work. Juan José Madrigal developed the mathematical analyses and descriptions from architectural and musical theories; he developed the description of the mathematical language. Both authors have developed the conclusions of the research.

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